Quantum hypergraph states

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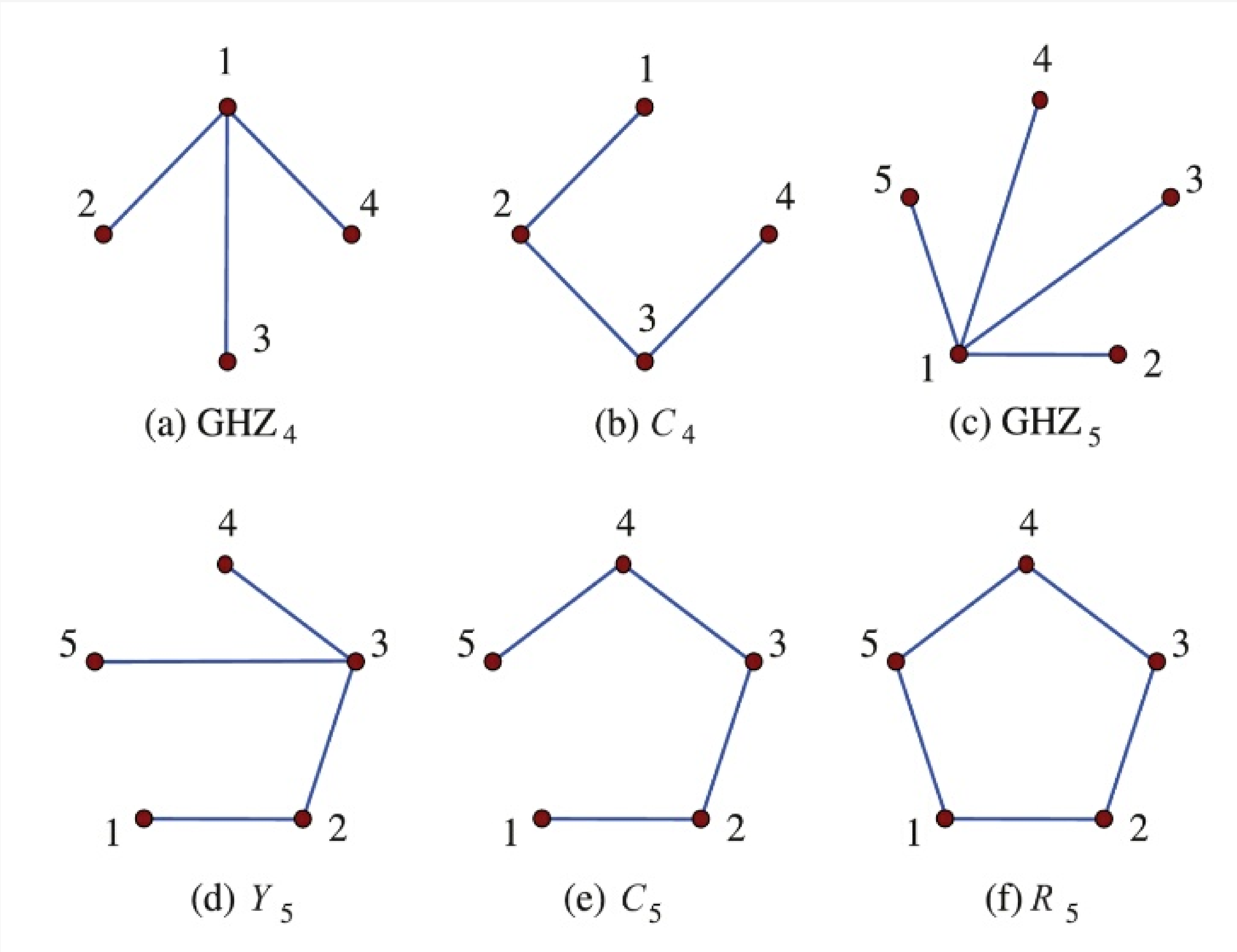
Structure

- Graph states & hypergraph states
- Entanglement properties of hyper-graph states
- Bell inequalities for HG states

Graph states



Stabilizers:

$$g_i = X_i \bigotimes_{j \in N(i)} Z_j$$

Eigenstate

$$|G\rangle = g_i |G\rangle$$

Alternative Definition

- Start with product state:

$$|\psi\rangle = |+\rangle|+\rangle|+\rangle \ldots\ldots |+\rangle$$

- Apply two-qubit gates for each edge:

$$|G\rangle = \prod_{ij \in E} C_{ij} |\psi\rangle$$

- mit $C_{ij} = \mathrm{CPHASE} = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -1 \end{pmatrix}$

Local equivalence

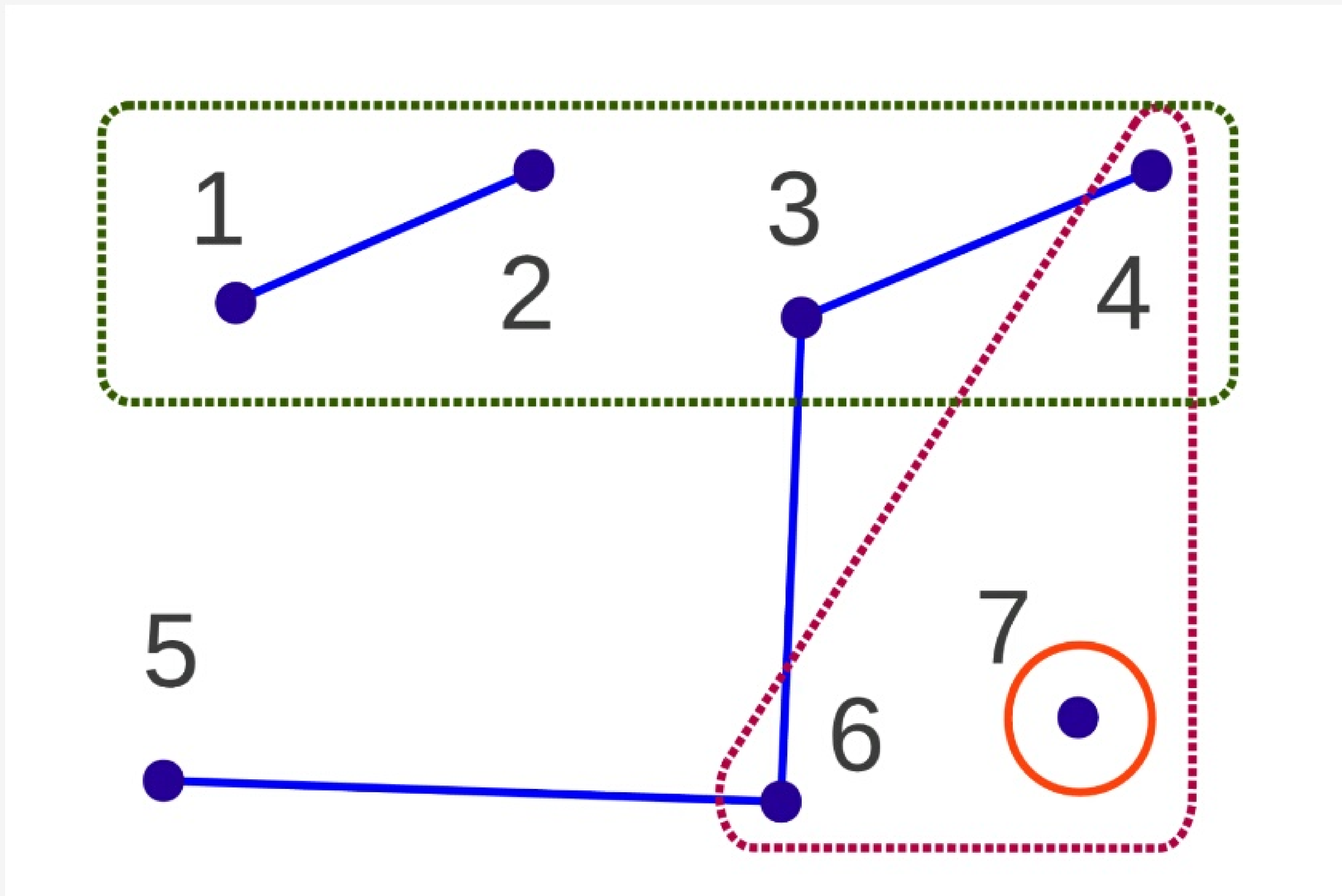
- Different graphs lead to equivalent states
- Possible local unitary ≈ local Clifford (but ≠)
- local Clifford = local complementation of the graph

Applications

- GHZ, cluster, etc : all are graph states
- Quantum error correction
- Bell inequalities : Mermin ...

Hypergraphs

Edges contain more than one vertex



Hypergraph states

- Start with a product state: $|\psi\rangle = |+\rangle|+\rangle \dots |+\rangle$
- Apply multi-qubit CPHASE for each edge:

$$|H\rangle = \prod_e C_e |\psi\rangle$$

- Example:

$$C_{123} = \begin{pmatrix} 1 &&&&&&& \\ & 1 &&&&&& \\ && 1 &&&&& \\ &&& 1 &&&& \\ &&&& 1 &&& \\ &&&&& 1 && \\ &&&&&& 1 & \\ &&&&&&& -1 \end{pmatrix}$$

The nonlocal stabilizer

- Define for each vertex:

$$g_i = X_i \prod_{\substack{e \text{ with} \\ i \in e}} C_{e \setminus i}$$

- Then: g_i generate Abelian group,

$$g_i |H\rangle = |H\rangle$$

Useful formulas

We have:

$$X_k C_e X_k = C_e C_{e \setminus \{k\}}$$

$$C_e X_k C_e = X_k \otimes C_{e \setminus \{k\}}$$

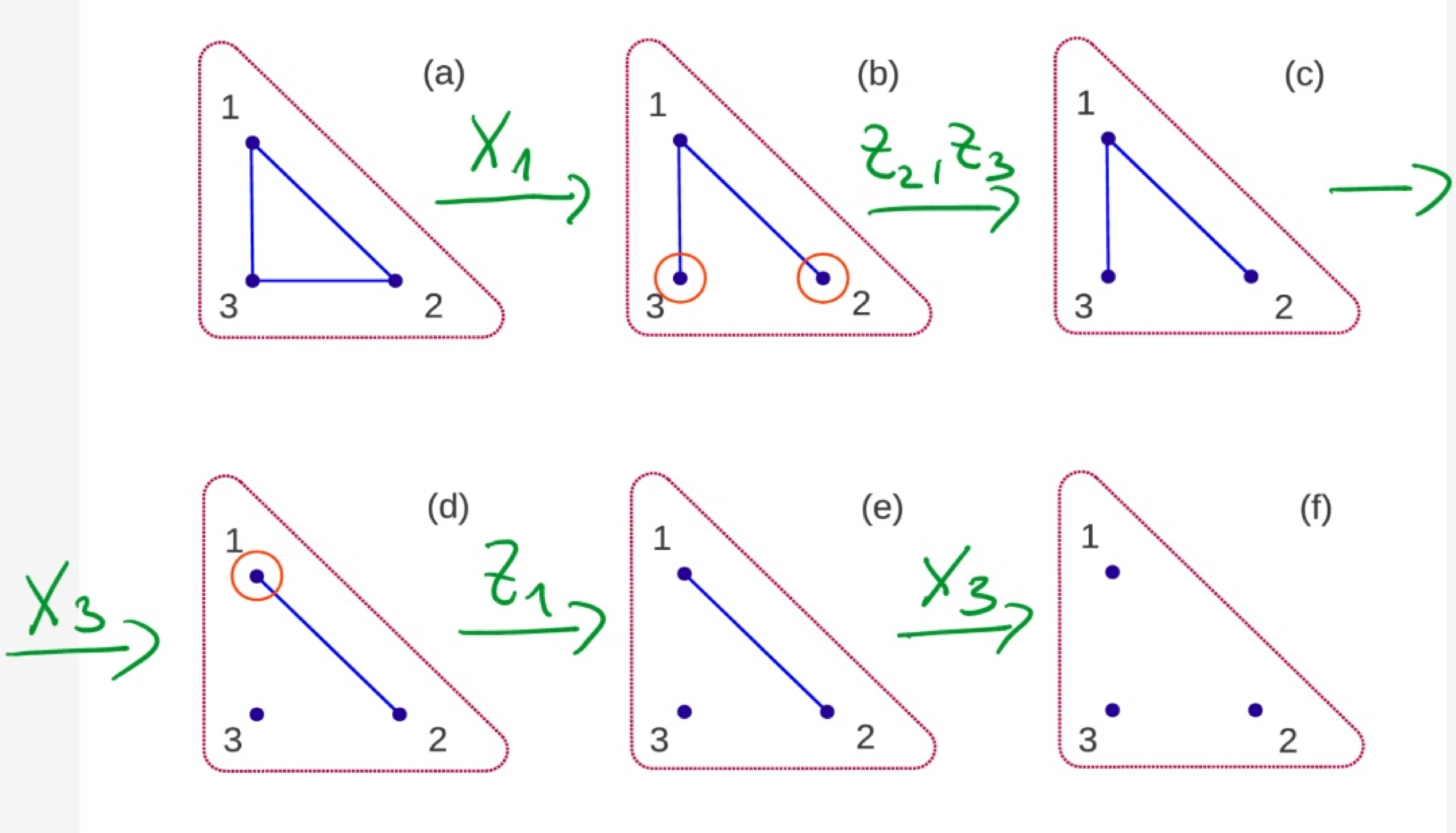
Since $C_\emptyset = -1$ and $C_k = Z_k$ this generalizes:

$$Z_k X_k = -X_k Z_k$$

Local Pauli transformations

- Transformation $Z_u |H\rangle \Rightarrow$ Add $e = \{u\}$
- Transformation $X_u |H\rangle \Rightarrow$ graphical rule $\Rightarrow$
- Transformation $Y_u |H\rangle \Rightarrow$ First Z_u, then X_u

An Example



Interesting Question:

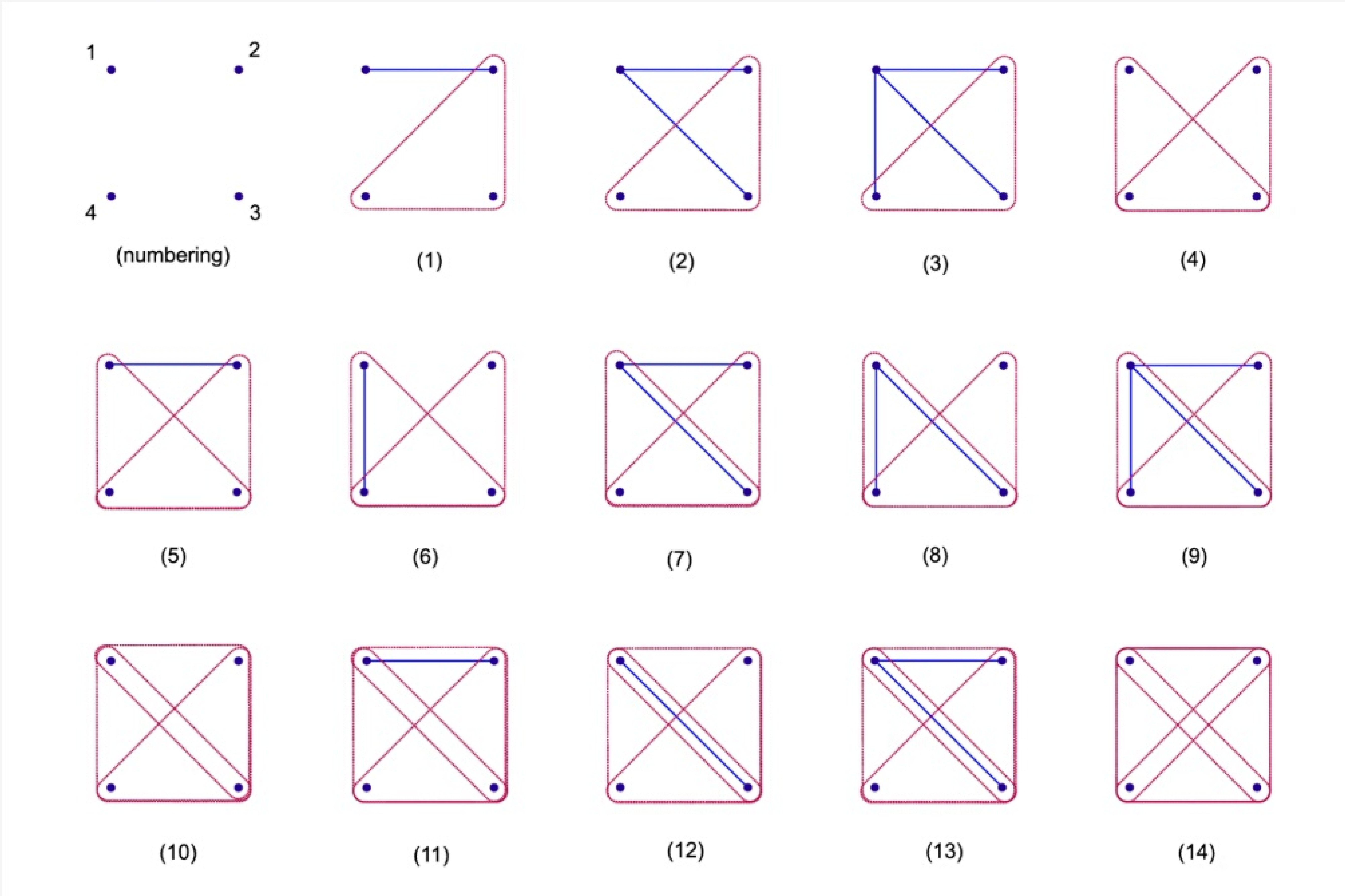
- Is for HG states

$$LU \Longleftrightarrow LP \quad ?$$

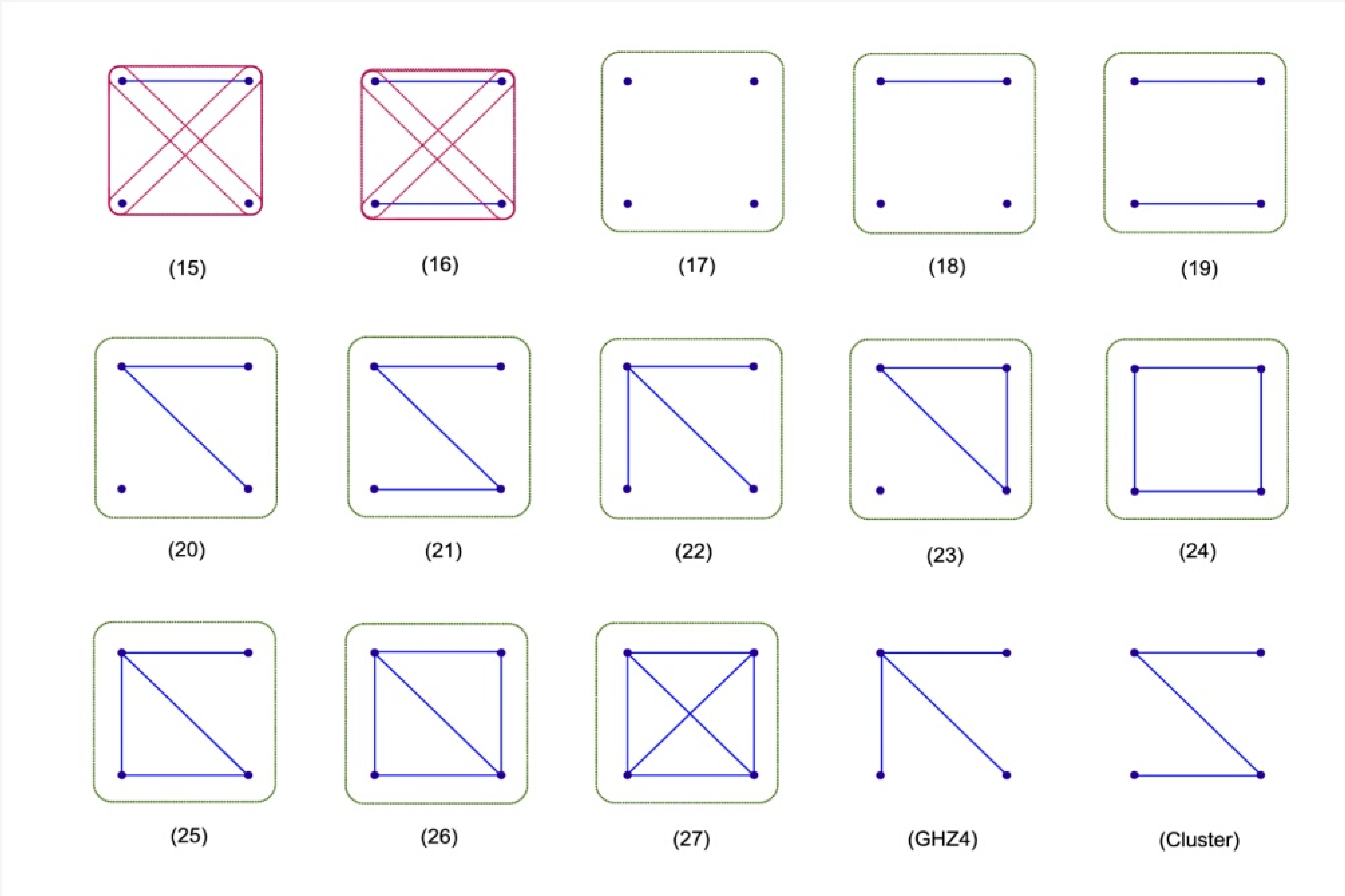
- For small N and typical states with large N this seems to be the case....

- But in general: unlikely....

Four qubits: 27 HG states

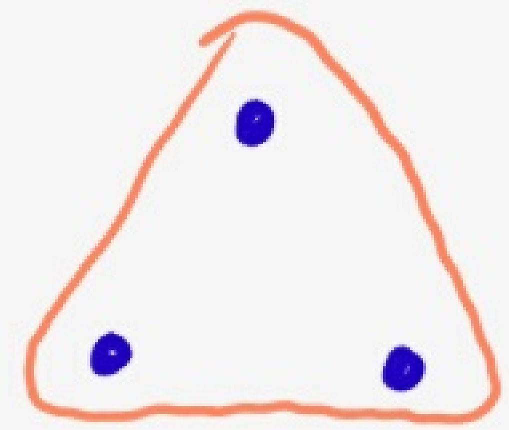


Four qubits: 27 HG states



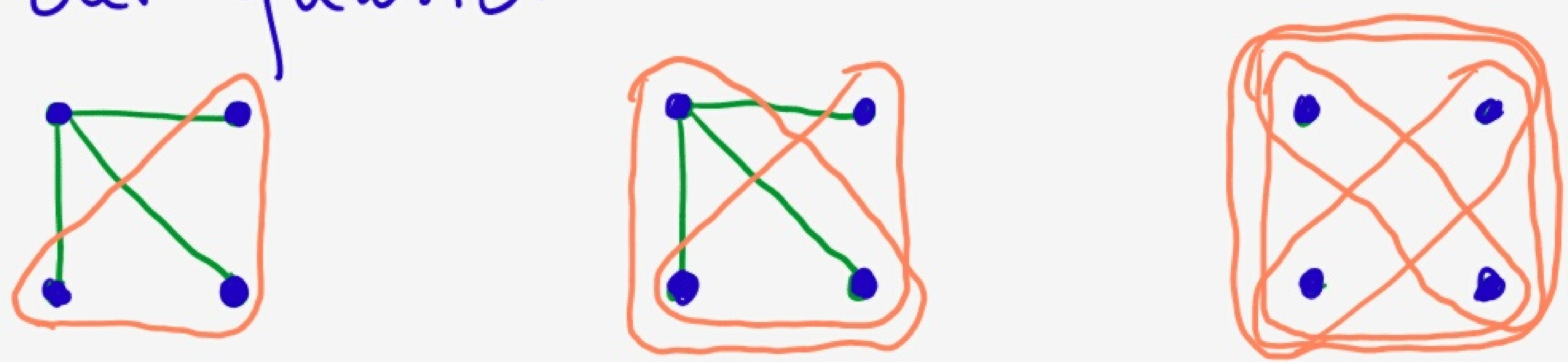
Interesting states

- Three qubits:

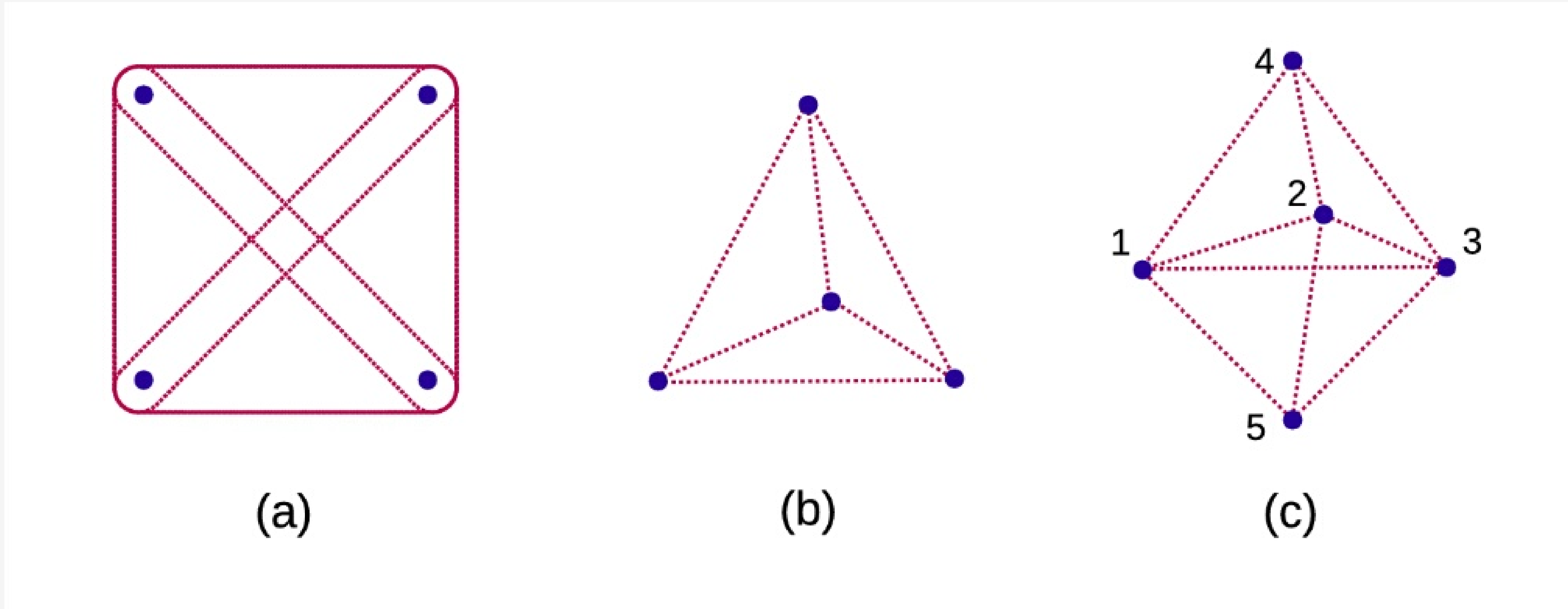


$$|H_3\rangle \sim |000\rangle + |010\rangle + |100\rangle + |111\rangle$$

- Four qubits:



have $g_i \sim \mathbb{1}$



Tetrahedron states for four and five qubits have maximally mixed single qubit reduced matrices.

Mermin & GHZ

Recall that:

$$M = g_1 + g_2 + g_3 + g_1 g_2 g_3$$

$$= XZZ + ZXZ + ZZX - XXX$$

$$= \begin{cases} 2 & \text{LHV} \\ 4 & \text{GHZ} \end{cases}$$

Idea for HG states

Find a k-uniform, fully connected N-qubit state with

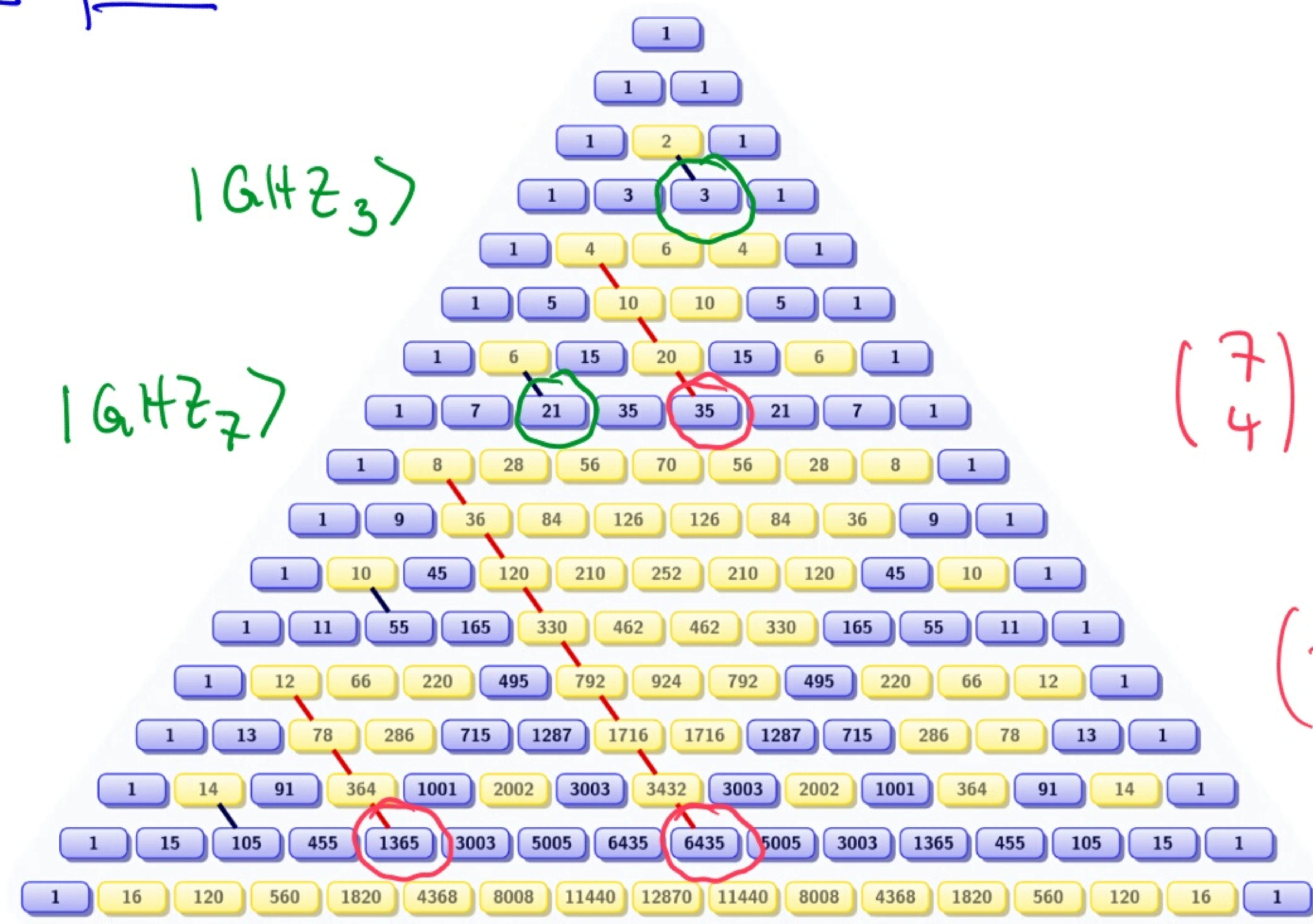
$$g_1 \cdot g_2 \cdot g_3 \cdots g_N = - X_1 X_2 X_3 \cdots X_N$$

This gives a Kochen Specker and Bell inequality.

Examples

$|GHZ_3\rangle$

$|GHZ_7\rangle$



$\binom{7}{4}$-state

$\binom{15}{4}$

$\binom{15}{8}$

Bell inequalities for 3 qubits

We have

$$X_1 C_{23} |H_3\rangle = |H_3\rangle$$

and

$$C_{23} = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -1 \end{pmatrix} \begin{array}{l} \left.\begin{array}{l} \\ \\ \\ \end{array}\right\} + \\ \} - \end{array}$$

Local correlations

This implies

$$P(+--\,|\,XZZ) = 0 \qquad \text{(a)}$$

$$P(-++\,|\,XZZ) + P(-+-\,|\,XZZ) + P(--+\,|\,XZZ) = 0 \qquad \text{(b)}$$

Lemma

Consider a biseparable LHV model $A|BC$ with non-signalling $B \leftrightarrow C$.
Assume the conditions (a), (b) + perm.
Then:

$$p(--+|XXX) = 0$$

HG state: $p(--+|XXX) = \frac{1}{16}$ ↯

Conclusions

- HG states are nice, because they can be described with a stabilizer.
- One can derive various Bell inequalities for them.

Ref: O. Gühne et al., JPA 47, 335303 (2014)