How long does it take to obtain a physical density matrix?

L. Knips[1,2] , C. Schwemmer[1,2] , N. Klein[1,2], J. Reuter[3], G. Tóth[4,5,6], and H. Weinfurter[1,2]

[1]Max-Planck-Institut für Quantenoptik, Garching, Germany
[2]Department für Physik, Ludwig-Maximilians-Universtät, München, Germany
[3]Bethe Center for Theoretical Physics, Universität Bonn, D-53115 Bonn, Germany
[4]Theoretical Physics, University of the Basque Country, Bilbao, Spain
[5]IKERBASQUE, Basque Foundation for Science, Bilbao, Spain
[6]Wigner Research Centre for Physics, Budapest, Hungary

ICOQI, Wien, Austria
11 December 2018


F.Y.R.O.M. - The Former Yugoslav Republic of Macedonia
Greenland (DENMARK)
Jan Mayen (NORWAY)
Greenland Sea
Denmark Strait
Barents Sea
Norwegian Sea
Arctic Circle
Reykjavík
ICELAND
Hammerfest
Tromsø
Murmansk
Kiruna
White Sea
Arkhangelsk
NORWAY
Luleå
Oulu
Umeå
FINLAND
Lake Onega
Trondheim
SWEDEN
Gulf of Bothnia
Lake Ladoga
Tampere
Tórshavn
Faroe Islands (DENMARK)
Helsinki
St. Petersburg
SHETLAND ISLANDS
Gävle
Turku
Gulf of Finland
Bergen
Oslo
ALAND ISLANDS
Tallinn
ESTONIA
RUSSIA
Rockall (U.K.)
ORKNEY ISLANDS
Stockholm
HEBRIDES
Stavanger
Göteborg
Gotland
LATVIA
Riga
North Atlantic Ocean
Aberdeen
North Sea
Baltic Sea
Öland
Glasgow
Edinburgh
DENMARK
Copenhagen
Malmö
LITHUANIA
Vilnius
Vitsyebsk
Smolensk
Belfast
UNITED KINGDOM
Kaliningrad
RUSSIA
Minsk
Mahilyow
Dublin
IRELAND
Irish Sea
Isle of Man (U.K.)
Leeds
Liverpool
Manchester
Bornholm
Gdańsk
BELARUS
Hrodna
Homyel
Hamburg
Bremen
Berlin
Poznań
Warsaw
Birmingham
Cardiff
London
Amsterdam
NETH.
POLAND
Łódź
Wrocław
Brest
Kiev
Rivne
Celtic Sea
Rotterdam
Essen
Cologne
Leipzig
Brussels
BEL.
Lille
Bonn
GERMANY
Prague
UKRAINE
L'viv
English Channel
Guernsey (U.K.)
Jersey (U.K.)
Frankfurt am Main
Luxembourg
LUX.
CZECH REPUBLIC
Brno
Kraków
Chernivtsi
SLOVAKIA
Chisinau
Paris
Stuttgart
Strasbourg
Munich
Bratislava
Vienna
AUSTRIA
Budapest
Cluj-Napoca
Iaşi
MOLDOVA
Nantes
LIECH.
HUNGARY
Zürich
Bern
SWITZ.
Geneva
Vaduz
ROMANIA
FRANCE
Bay of Biscay
Lyon
Ljubljana
SLOVENIA
Zagreb
Bucharest
A Coruña
Bordeaux
Milan
Venice
Turin
CROATIA
BOSNIA AND HERZEGOVINA
Belgrade
Varna
Bilbao
Genoa
Sarajevo
SERBIA AND MONTENEGRO
BULGARIA
Toulouse
MONACO
SAN MARINO
Florence
Sofia
Porto
Andorra la Vella
ANDORRA
Marseille
Ligurian Sea
ITALY
Adriatic Sea
Skopje
F.Y.R.O.M.
Istanbul
Zaragoza
Corsica
Rome
Tirana
ALB.
PORTUGAL
Madrid
Barcelona
VATICAN CITY
Thessaloniki
Lisbon
Balearic Sea
Naples
SPAIN
Valencia
Sardinia
Tyrrhenian Sea
GREECE
Aegean Sea
Athens
BALEARIC ISLANDS
Sevilla
Cagliari
Ionian Sea
Mediterranean Sea
Palermo
Gibraltar (U.K.)
Málaga
Sicily
Strait of Gibraltar
Ceuta (SPAIN)
Alborán Sea
Melilla (SPAIN)
Oran
Algiers
Tunis
Crete
Rabat
Casablanca
MOROCCO
ALGERIA
TUNISIA
Valletta
MALTA
Scale 1: 19,500,000
Lambert Conformal Conic Projection, standard parallels 40°N and 56°N
300 Kilometers
300 Miles

Outline

1. **Motivation**
   - Why quantum tomography is important?
2. **Quantum experiments with multi-qubit systems**
   - Physical systems
   - Local measurements
3. **Full quantum state tomography**
   - Basic ideas and scaling
   - Approaches to solve the scalability problem
   - Experiments
4. **Obtaining a physical density matrix**
   - The usual fitting procedure to obtain a physical matrix
   - Our insight: the key are the eigenvalues
   - Our proposal: Hypothesis testing

Why tomography is important?

- Many experiments are aiming to create many-body entangled states.
- Quantum state tomography can be used to check how well the state has been prepared.
- However, there are some long standing problems:
  - the number of measurements scales exponentially with the number of qubits,
  - the “raw” density matrices obtained from state tomography is not physical, etc.

Outline

1. **Motivation**
   - Why quantum tomography is important?
2. **Quantum experiments with multi-qubit systems**
   - Physical systems
   - Local measurements
3. **Full quantum state tomography**
   - Basic ideas and scaling
   - Approaches to solve the scalability problem
   - Experiments
4. **Obtaining a physical density matrix**
   - The usual fitting procedure to obtain a physical matrix
   - Our insight: the key are the eigenvalues
   - Our proposal: Hypothesis testing

Physical systems

Examples

- 14 qubits with trapped cold ions
  T. Monz, P. Schindler, J.T. Barreiro, M. Chwalla, D. Nigg, W.A. Coish, M. Harlander, W. Haensel, M. Hennrich, R. Blatt, Phys. Rev. Lett. 106, 130506 (2011).

- 10 qubits with photons
  W.-B. Gao, C.-Y. Lu, X.-C. Yao, P. Xu, O. Gühne, A. Goebel, Y.-A. Chen, C.-Z. Peng, Z.-B. Chen, J.-W. Pan, Nature Physics, 6, 331 (2010).

Outline

1. Motivation
   - Why quantum tomography is important?
2. Quantum experiments with multi-qubit systems
   - Physical systems
   - Local measurements
3. Full quantum state tomography
   - Basic ideas and scaling
   - Approaches to solve the scalability problem
   - Experiments
4. Obtaining a physical density matrix
   - The usual fitting procedure to obtain a physical matrix
   - Our insight: the key are the eigenvalues
   - Our proposal: Hypothesis testing

Only local measurements are possible

Definition

A single local measurement setting is the basic unit of experimental effort.

A local setting means measuring operator $A^{(k)}$ at qubit k for all qubits.

$A^{(1)}$	$A^{(2)}$	$A^{(3)}$	...	$A^{(n)}$

- All two-qubit, three-qubit correlations, etc. can be obtained.

$$\langle A^{(1)} A^{(2)} \rangle, \langle A^{(1)} A^{(3)} \rangle, \langle A^{(1)} A^{(2)} A^{(3)} \rangle \ldots$$

Outline

1. **Motivation**
   - Why quantum tomography is important?
2. **Quantum experiments with multi-qubit systems**
   - Physical systems
   - Local measurements
3. **Full quantum state tomography**
   - Basic ideas and scaling
   - Approaches to solve the scalability problem
   - Experiments
4. **Obtaining a physical density matrix**
   - The usual fitting procedure to obtain a physical matrix
   - Our insight: the key are the eigenvalues
   - Our proposal: Hypothesis testing

Full quantum state tomography

- The density matrix can be reconstructed from 3^n measurement settings ("overcomplete tomography").

Example

For $n = 4$, the measurements are

1.	X	X	X	X
2.	X	X	X	Y
3.	X	X	X	Z
		...		
3^4.	Z	Z	Z	Z

- Note again that the number of measurements scales exponentially in n.

Outline

1. **Motivation**
   - Why quantum tomography is important?
2. **Quantum experiments with multi-qubit systems**
   - Physical systems
   - Local measurements
3. **Full quantum state tomography**
   - Basic ideas and scaling
   - Approaches to solve the scalability problem
   - Experiments
4. **Obtaining a physical density matrix**
   - The usual fitting procedure to obtain a physical matrix
   - Our insight: the key are the eigenvalues
   - Our proposal: Hypothesis testing

Approaches to solve the scalability problem

- If the state is expected to be of a certain form (MPS), we can measure the parameters of the ansatz.
  S.T. Flammia *et al.,* arxiv:1002.3839; M. Cramer, M.B. Plenio, arxiv:1002.3780; O. Landon-Cardinal *et al.,* arxiv:1002.4632.

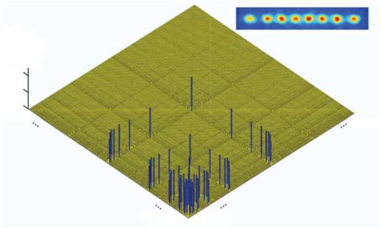
- If the state is of low rank, we need fewer measurements.
  D. Gross et al., Phys. Rev. Lett. 105, 150401 (2010).

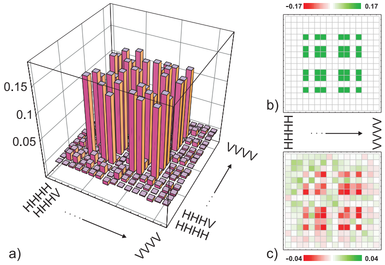
- Permutationally invariant tomography, tomography in a subspace of the density matrices.
  G. Tóth *et al.* , Phys. Rev. Lett. 105, 250403 (2010); T. Moroder *et al.*, New J. Phys. 14, 105001 (2012); C. Schwemmer *et al.*, Phys. Rev. Lett. 113, 040503 (2014)

Outline

1. **Motivation**
   - Why quantum tomography is important?
2. **Quantum experiments with multi-qubit systems**
   - Physical systems
   - Local measurements
3. **Full quantum state tomography**
   - Basic ideas and scaling
   - Approaches to solve the scalability problem
   - Experiments
4. **Obtaining a physical density matrix**
   - The usual fitting procedure to obtain a physical matrix
   - Our insight: the key are the eigenvalues
   - Our proposal: Hypothesis testing

Experiments with ions and photons





- 8 ions: H. Haeffner, W. Haensel, C. F. Roos, J. Benhelm, D. Chek-al-kar, M. Chwalla, T. Koerber, U. D. Rapol, M. Riebe, P. O. Schmidt, C. Becher, O. Gühne, W. Dür, R. Blatt, Nature 438, 643-646 (2005).
- 4 photons: N. Kiesel, C. Schmid, G. Tóth, E. Solano, and H. Weinfurter, Phys. Rev. Lett. 98, 063604 (2007).
- 6 photons: C. Schwemmer, G. Tóth, A. Niggebaum, T. Moroder, D. Gross, O. Gühne, and H. Weinfurter, Phys. Rev. Lett. 113, 040503 (2014).

Outline

1. **Motivation**
   - Why quantum tomography is important?
2. **Quantum experiments with multi-qubit systems**
   - Physical systems
   - Local measurements
3. **Full quantum state tomography**
   - Basic ideas and scaling
   - Approaches to solve the scalability problem
   - Experiments
4. **Obtaining a physical density matrix**
   - The usual fitting procedure to obtain a physical matrix
   - Our insight: the key are the eigenvalues
   - Our proposal: Hypothesis testing

Obtain a density matrix

- The density matrix can be decomposed into correlations as

$$\varrho = \frac{1}{2^n} \sum_{\boldsymbol{\mu}} T_{\boldsymbol{\mu}} \sigma_{\boldsymbol{\mu}},$$

  where

$$\sigma_{\boldsymbol{\mu}} = \sigma_{\mu_1} \otimes \sigma_{\mu_2} \otimes \cdots \otimes \sigma_{\mu_n},$$

  $\mu_i \in \{0, 1, 2, 3\}$.

- $\sigma_{1/2/3}$ are Pauli spin matrices, and σ_0 is the identity matrix.

- The correlation matrix is defined as

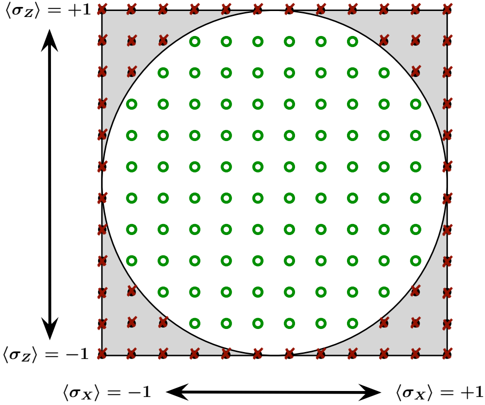
$$T_{\boldsymbol{\mu}} = \langle \sigma_{\boldsymbol{\mu}} \rangle.$$

Obtain a density matrix II

- How can we obtain the estimate $\tilde{\varrho}$? We just measure $T_{\boldsymbol{\mu}} = \langle \sigma_{\boldsymbol{\mu}} \rangle$. (linear inversion)
- Problem: we have finite number of measurements.
- Hence, we cannot get $\langle \sigma_{\boldsymbol{\mu}} \rangle$ exactly and our state will not be physical.
- The consequence is not only a small error, but that $\varrho \geq 0$ is not fulfilled.

Obtain a density matrix III

- 1 qubit, 11 measurements.



[R. Blume-Kohout, arXiv:quant-ph/0611080]

Obtain a density matrix IV

The negative eigenvalues are

- due to finite statistics,
- are not due to some experimental error.

Why negative eigenvalues are a problem?

We cannot calculate

- fidelities with a mixed state,
- entropies,
- purity,
- entanglement, etc.

We can still calculate

- the fidelity with a pure state (=expectation value of a projector).

Fitting

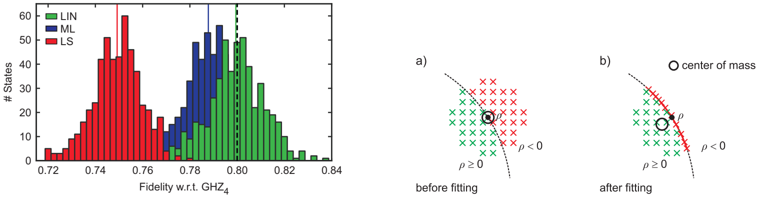
- Method to get rid of the negative eigenvalues of ϱ.
- Find the physical density matrix in a best agreement with the experimental data.
- Main methods: an elegant theory using maximum likelihood, least squares.

[Z. Hradil, Phys. Rev. A 55, R1561 (1997);
D. F. V. James, P. G. Kwiat, W. J. Munro, and A. G. White,
Phys. Rev. A 64, 052312 (2001);
R. Blume- Kohout, New J. Phys. 12, 043034 (2010).]

Problems with fitting

- Bias: ML is unbiased for infinite statistics. We have finite data.

- Fidelity changes, detection of fake entanglement



- Unfortunately, the fidelity with respect to the pure state we wanted to prepare also changes.

- Note that the variance decerased due to fitting!

[Schwemmer et al., PRL 114, 080403 (2015).]

Problems with fitting II

- Why are there problems?

- Not possible to have a fitting algorithm that is unbiased **and** gives a physical estimate.

- Example: prrepare a pure state $|\Psi\rangle$. Make tomography M times and construct the estimates $\tilde{\varrho}_k$.

- Then, the estimator is unbiased if

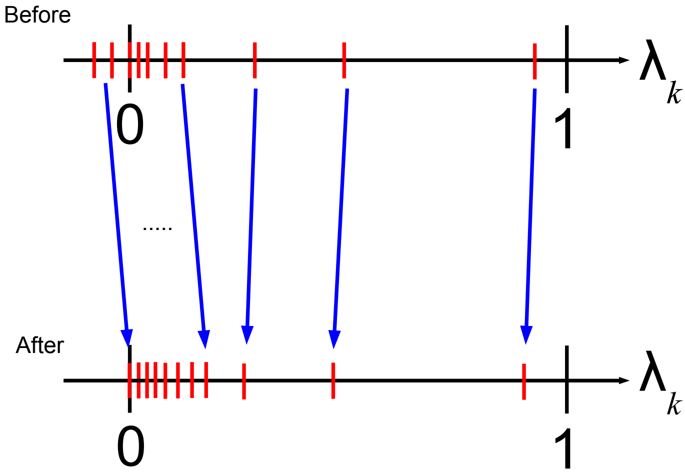
$$\frac{1}{M}\sum_{k=1}^{M}\tilde{\varrho}_k = |\Psi\rangle\langle\Psi|.$$

- Impossible, if $\tilde{\varrho}_k$ are physical.

  Derivation from
  [Schwemmer, Knips, Richart, Weinfurter, Moroder, Kleinmann, Gühne, PRL 114, 080403 (2015).]

Problems with fitting III

Another artifact: About half of the eigenvalues become zero.



Small eigenvalues increase   Large eigenvalues decrease

Heuristic explanation of the problem

Fitting is

- Like frequency analysis.
- Large frequencies do not matter if we have finite data.
  (Harald Weinfurter)

Heuristic explanation of the problem II

Fitting is

- Like the police is looking for someone, and the search warrant says that the person is 3 meter tall. Who is the closest in the database?
- Strategy 1: we look for a person who is the closest, i.e., very tall.
- Comment: we have to look for minimal distance from an impossible data set.
- Strategy 2: we ignore the useless data on how tall the person is, and take into account only other parameters.

  (GT)

Outline

1. **Motivation**
   - Why quantum tomography is important?
2. **Quantum experiments with multi-qubit systems**
   - Physical systems
   - Local measurements
3. **Full quantum state tomography**
   - Basic ideas and scaling
   - Approaches to solve the scalability problem
   - Experiments
4. **Obtaining a physical density matrix**
   - The usual fitting procedure to obtain a physical matrix
   - Our insight: the key are the eigenvalues
   - Our proposal: Hypothesis testing

Let us analyze the problem

- Completely mixed state

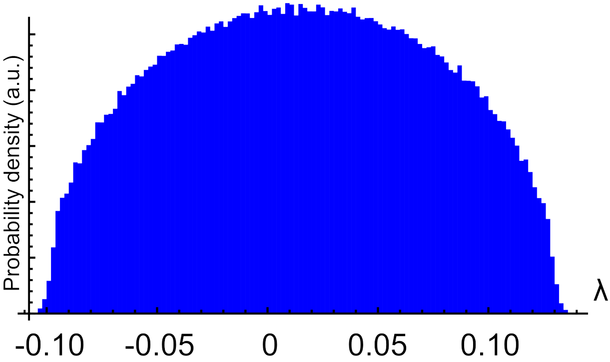
$$\varrho_{\rm wn} = \frac{1}{2^n}\sigma_{0,0,\ldots,0} = \frac{1}{2^n}\mathbb{1}$$

  with 2^n degenerate eigenvalues $\lambda_i = 1/2^n$.

- We use overcomplete tomography, which is based on measuring the Pauli correlations.

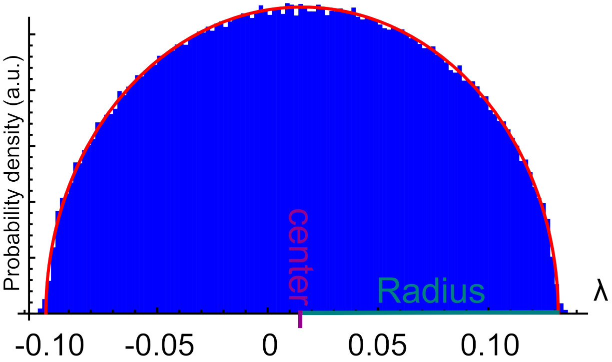
Distribution of eigenvalues

- Consider $n = 6$ qubit maximally mixed state
- Simulate $N = 100$ measurements per setting
- Estimate density matrix
- Repeat 10000 times
- Histogram of eigenvalues



Distribution of eigenvalues

- Consider $n = 6$ qubit maximally mixed state
- Simulate $N = 100$ measurements per setting
- Estimate density matrix
- Repeat 10000 times
- Histogram of eigenvalues



Distribution of eigenvalues II

The Wigner semicircle appears also in random matrix theory for a special type of matrices

- all elements have the same Gaussian distribution,
- they are not correlated with each other.

Now the elements of the density matrix obtained from tomography

- have different variances,
- they are even correlated with each other.

Highly non-trivial result: We prove that we obtain the Wigner semicircle distribution for the usual, overcomplete tomography.

Derivation (slide from Lukas Knips)

- Concept: compare moments of eigenvalue distribution to moments of ideal semicircle function

- Define semicircle distribution

$$f_{c,R}(x) = \underbrace{\frac{2}{\pi R^2}}_{Z} \sqrt{(x-c)^2 - R^2}$$

with (even) moments

$$m_2^{\mathrm{sc}} = \int_{-\infty}^{\infty} f_{0,R}(x)\, x^2 x = \left(\frac{R}{2}\right)^2,$$
$$m_4^{\mathrm{sc}} = \int_{-\infty}^{\infty} f_{0,R}(x)\, x^4 x = 2\left(\frac{R}{2}\right)^4,$$
$$m_6^{\mathrm{sc}} = \int_{-\infty}^{\infty} f_{0,R}(x)\, x^6 x = 5\left(\frac{R}{2}\right)^6,$$
$$m_8^{\mathrm{sc}} = \int_{-\infty}^{\infty} f_{0,R}(x)\, x^8 x = 14\left(\frac{R}{2}\right)^8.$$

Using the *Catalan* numbers

$$\mathcal{C}_{j+1} = \mathcal{C}_j \frac{2(2j+1)}{j+2}$$

we obtain

$$m_{2k}^{\mathrm{sc}} = \int_{-\infty}^{\infty} f_{0,R}(x)\, x^{2k} x = \mathcal{C}_k \left(\frac{R}{2}\right)^{2k}$$

- Odd (centralized) moments vanish

- Goal: reproduce Catalan numbers in distribution of eigenvalues

- Calculate all moments of eigenvalue distribution:

$$\begin{aligned}
m_k^{\mathrm{ev}} &= \frac{1}{2^n}\sum_{i=1}^{2^n} \mathbb{E}\left[\lambda_i^k\right] \\
&= \frac{1}{2^n}\mathbb{E}\left[\sum_{i=1}^{2^n} \lambda_i^k\right] \\
&= \mathbb{E}\left[\frac{1}{2^n}\operatorname{Tr}\left(D^k\right)\right] \\
&= \mathbb{E}\left[\frac{1}{2^n}\operatorname{Tr}\left(\left(U^\dagger \varrho U\right)^k\right)\right] \\
&= \mathbb{E}\left[\frac{1}{2^n}\operatorname{Tr}\left(\varrho^k\right)\right]
\end{aligned}$$

- Second moment of (centered) distribution:

$$\begin{aligned}
m_2^{\mathrm{ev}} &= \frac{1}{2^{3n}}\sum_{\vec\mu,\vec\nu} \mathbb{E}\left[T_{\vec\mu} T_{\vec\nu}\right] 2^n \delta_{\vec\mu,\vec\nu} \\
&= \frac{2^n}{2^{3n}}\sum_{\vec\mu} \mathbb{E}\left[T_{\vec\mu}^2\right]
\end{aligned}$$

overcomplete Pauli scheme:

$$\begin{aligned}
m_2^{\mathrm{ev}} &= \frac{1}{4^n N}\sum_{j=0}^{n-1}\binom{n}{j}\frac{3^{n-j}}{3^j} \\
&= \frac{10^n - 1}{12^n}\frac{1}{N}.
\end{aligned}$$

with n qubits, N events per basis element.

- Comparision of m_2^{sc}, m_2^{ev} yields:

$$R = 2\sqrt{\frac{10^n - 1}{12^n}}\frac{1}{\sqrt{N}}$$

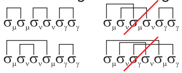
- Fourth moment:

$$\begin{aligned}
m_4^{\mathrm{ev}} &= \frac{1}{2^n}\sum_{i=1}^{2^n}\mathbb{E}\left[\lambda_i^4\right] \\
&= \frac{1}{2^{5n}}\sum_{\vec\mu,\vec\nu,\vec\gamma,\vec\lambda}\mathbb{E}\left[T_{\vec\mu}T_{\vec\nu}T_{\vec\gamma}T_{\vec\lambda}\right] \\
&\qquad \cdot \operatorname{Tr}\left(\sigma_{\vec\mu}\sigma_{\vec\nu}\sigma_{\vec\gamma}\sigma_{\vec\lambda}\right) \\
&= \frac{1}{2^{5n}}\frac{1}{2!}\sum_{\vec\mu}\sum_{\vec\nu:\{\vec\nu\neq\vec\mu\}}\mathbb{E}\left[T_{\vec\mu}^2 T_{\vec\nu}^2\right] \\
&\qquad \cdot \operatorname{Tr}\left(\sum_{i=1}^{6}\mathcal{P}_i\left(\sigma_{\vec\mu}\sigma_{\vec\mu}\sigma_{\vec\nu}\sigma_{\vec\nu}\right)\right)
\end{aligned}$$

- Sixth moment:

$$\begin{aligned}
m_6^{\mathrm{ev}} &= \frac{1}{2^n}\sum_{i=1}^{2^n}\mathbb{E}\left[\lambda_i^6\right] \\
&\approx \frac{1}{2^{7n}}\frac{1}{3!}\sum_{\vec\mu}\sum_{\vec\nu:\{\vec\nu\neq\vec\mu\}}\sum_{\vec\gamma:\{\vec\gamma\neq\vec\nu,\vec\gamma\neq\vec\mu\}} \\
&\qquad \mathbb{E}\left[T_{\vec\nu}^2 T_{\vec\mu}^2 T_{\vec\gamma}^2\right] \cdot \\
&\qquad \operatorname{Tr}\left(\sum_{i=1}^{90}\mathcal{P}_i\left(\sigma_{\vec\mu}\sigma_{\vec\mu}\sigma_{\vec\nu}\sigma_{\vec\nu}\sigma_{\vec\gamma}\sigma_{\vec\gamma}\right)\right)
\end{aligned}$$

non-crossing crossing

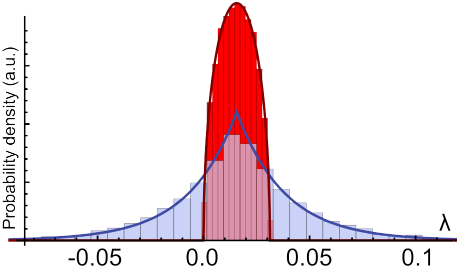


- Only non-crossing partitions (amount given by *Catalan* numbers) contribute:

$$m_k^{\mathrm{ev}} = \frac{1}{2^n}\sum_{i=1}^{2^n}\mathbb{E}\left[\lambda_i^{2k}\right] = \frac{\mathcal{C}_k}{N^k} \quad \square$$

Other type of tomography

- Not all tomographies lead to a Wigner semicircle



[F. V. James, P. G. Kwiat, W. J. Munro, and A. G. White, Phys. Rev. A 64, 052312 (2001).]

How long do we have to measure to get a physical state?

- Pure state mixed with white noise

$$\varrho_q = q|\psi\rangle\langle\psi| + (1-q)\varrho_{\mathrm{cm}}.$$

- The center of the semicircle is shifted to

$$c_q = \frac{1-q}{2^n - r}.$$

- The radius of the semicircle is

$$R = 2\sqrt{\frac{10^n - 1}{12^n}}\frac{1}{\sqrt{N}} \approx 2\left(\frac{5}{6}\right)^{\frac{n}{2}}\frac{1}{\sqrt{N}}.$$

- Physical ϱ if

$$R \leq c_q \Rightarrow N \geq N_0 = 4\left(\frac{5}{6}\right)^n \left(\frac{2^n - 1}{1-q}\right)^2.$$

How long do we have to measure to get a physical state? II

- The minimum number of measurements needed is

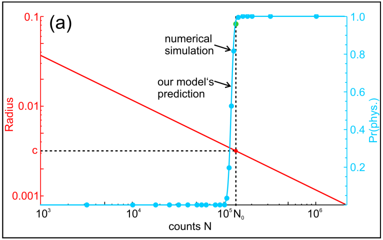
$$N_0 = 4\left(\frac{5}{6}\right)^n \left(\frac{2^n - 1}{1 - q}\right)^2,$$

  where n is the number of qubits.

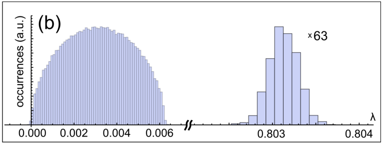
- The limit on the number of measurements is sharp, since we have a Wigner semicircle distribution.

How long do we have to measure to get a physical state? III

- Six-qubit GHZ state mixed with $q = 0.2$ white noise, radius and probability of physical matrix



- The eigenvalues of the noisy state at $N = N_0$ measurements



Outline

1. **Motivation**
   - Why quantum tomography is important?
2. **Quantum experiments with multi-qubit systems**
   - Physical systems
   - Local measurements
3. **Full quantum state tomography**
   - Basic ideas and scaling
   - Approaches to solve the scalability problem
   - Experiments
4. **Obtaining a physical density matrix**
   - The usual fitting procedure to obtain a physical matrix
   - Our insight: the key are the eigenvalues
   - Our proposal: Hypothesis testing

Hypothesis testing

- Assume
$$\varrho = p\varrho_r + (1-p)\frac{\mathbb{1}}{2^n},$$
where ϱ_r is a pure of a low rank state.

- Colored noise? We cannot tell, since the Wigner semicircle will cover all structure.

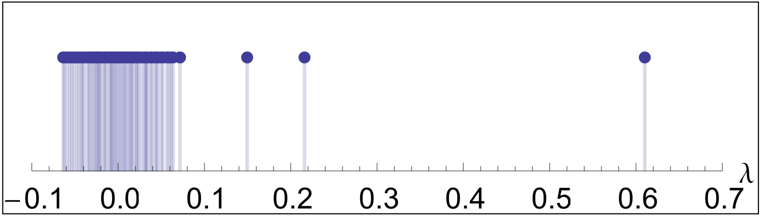
- Hypothesis about how many eigenvalues are “real”.

Hypothesis testing

- We prepare experimentally a six-qubit Dicke state

$$|D_6^{(3)}\rangle = \frac{1}{\sqrt{6}}(|000111\rangle + |001011\rangle + ... + |111000\rangle).$$

- Quantum state tomography with around 230 events per setting.
- Hypothesis: 3 eigenvalues + noise. Is this correct?

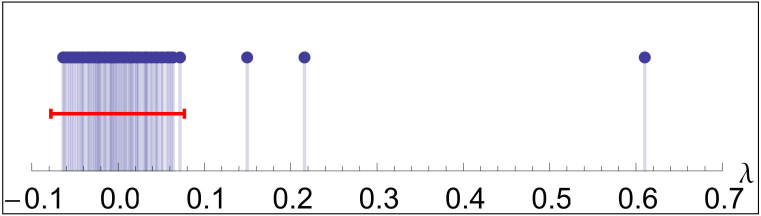


Hypothesis testing II

- We prepare experimentally a six-qubit Dicke state

$$|D_6^{(3)}\rangle = \frac{1}{\sqrt{6}}(|000111\rangle + |001011\rangle + ... + |111000\rangle).$$

- Quantum state tomography with around 230 events per setting.
- Hypothesis: 3 eigenvalues + noise. Is this correct?

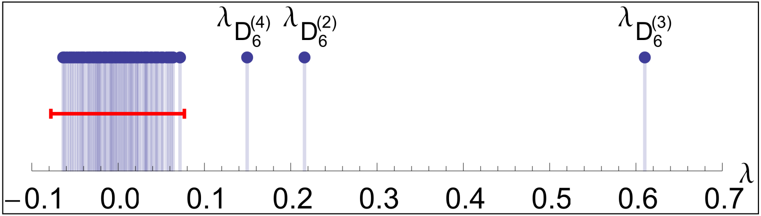


Hypothesis testing III

- We prepare experimentally a six-qubit Dicke state

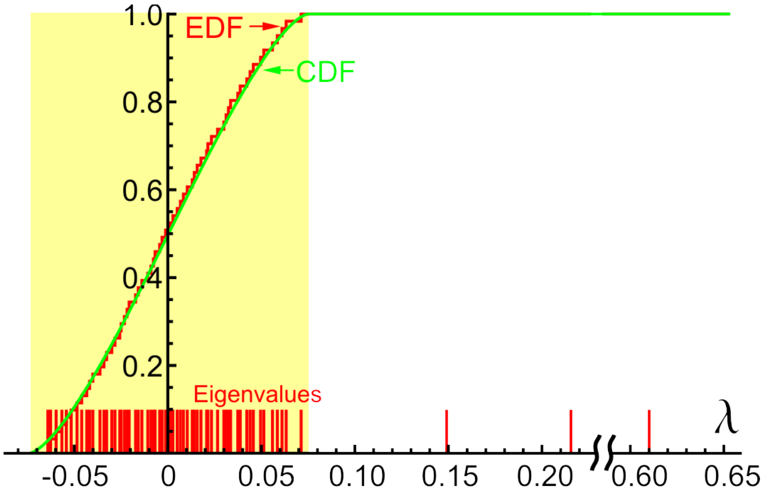
$$|D_6^{(3)}\rangle = \frac{1}{\sqrt{6}}(|000111\rangle + |001011\rangle + ... + |111000\rangle).$$

- Quantum state tomography with around 230 events per setting.
- Hypothesis: 3 eigenvalues + noise. Is this correct?



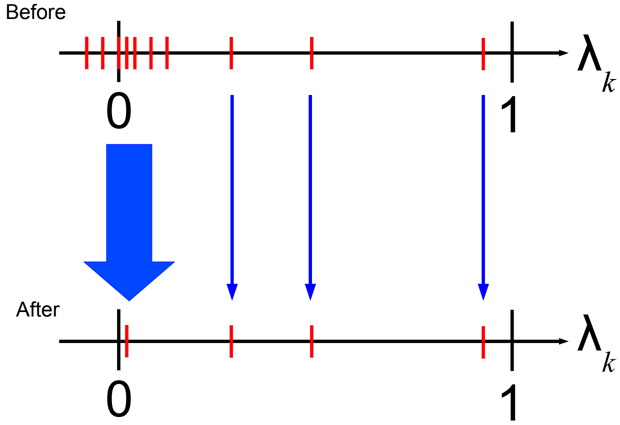
Is the hypothesis correct?

- Empirical distribution function (EDF) vs. Cumulative distribution function (CDF) of the Wigner semicircle



Our method in a single figure

Large, useful eigenvalues are not affected!

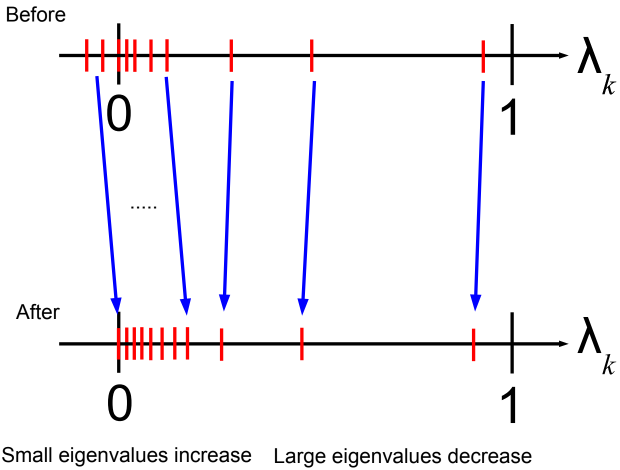


Small eigenvalues are replaced by their average

Large eigenvalues do not change

Just to compare: old method

Large, useful eigenvalues are affected!



Summary

- We discussed the distribution of the eigenvalues of density matrices obtained from tomography.
- We proposed a simple solution for a long standing problem, namely, getting rid of the negative eigenvalues.
- I thank Lukas Knips for most of the figures for this talk.

See:

L. Knips, C. Schwemmer, N. Klein, J. Reuter, G. Tóth, and H. Weinfurter,

How long does it take to obtain a physical density matrix?, arxiv:1512.06866.

THANK YOU FOR YOUR ATTENTION!

erc European Research Council

MINISTERIO DE CIENCIA E INNOVACIÓN

EUSKO JAURLARITZA GOBIERNO VASCO