Detecting metrologically useful multipartite entanglement in cold gases

Géza Tóth[1,2,3,4,5]

[1]University of the Basque Country UPV/EHU, Bilbao, Spain
[2]EHU Quantum Center, University of the Basque Country UPV/EHU, Spain
[3]Donostia International Physics Center (DIPC), San Sebastián, Spain
[4]IKERBASQUE, Basque Foundation for Science, Bilbao, Spain
[5]Wigner Research Centre for Physics, Budapest, Hungary

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Oxford, 28 March 2023

F.Y.R.O.M. - The Former Yugoslav Republic of Macedonia
Greenland (DENMARK)
Jan Mayen (NORWAY)
Greenland Sea
Denmark Strait
Barents Sea
Hammerfest
Tromsø
Murmansk
Norwegian Sea
Reykjavík
ICELAND
Arctic Circle
White Sea
Arkhangel
Kiruna
Luleå
Oulu
NORWAY
Umeå
FINLAND
Lake Onega
Trondheim
SWEDEN
Gulf of Bothnia
Tampere
Lake Ladoga
Tórshavn
Faroe Islands (DENMARK)
Gävle
Turku
Helsinki
St. Petersburg
SHETLAND ISLANDS
Bergen
Oslo
ALAND ISLANDS
Gulf of Finland
Tallinn
ESTONIA
RUSSIA
Rockall (U.K.)
ORKNEY ISLANDS
Stockholm
Stavanger
HEBRIDES
Aberdeen
Göteborg
Gotland
LATVIA
Riga
North Atlantic Ocean
Glasgow
Edinburgh
North Sea
DENMARK
Copenhagen
Baltic Sea
Öland
LITHUANIA
Vilnius
Vitsyebsk
Smolensk
Belfast
UNITED
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RUSSIA
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Lyon
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Milan
Venice
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A Coruña
Bordeaux
Turin
CROATIA
Belgrade
Varna
Bilbao
BOSNIA AND HERZEGOVINA
Genoa
MONACO
SAN MARINO
Sarajevo
SERBIA AND MONTENEGRO
BULGARIA
Toulouse
Sofia
Andorra la Vella
Marseille
Florence
Ligurian Sea
Adriatic Sea
Istanbul
Porto
ANDORRA
ITALY
Skopje
Zaragoza
Corsica
Rome
F.Y.R.O.M.
Tirana
Thessaloniki
PORTUGAL
Madrid
Barcelona
VATICAN CITY
ALB.
Lisbon
SPAIN
Valencia
Balearic Sea
Naples
TURK
Sardinia
Tyrrhenian Sea
GREECE
Aegean Sea
Sevilla
BALEARIC ISLANDS
Cagliari
Athens
Ionian Sea
Mediterranean Sea
Palermo
Gibraltar (U.K.)
Málaga
Sicily
Strait of Gibraltar
Ceuta (SPAIN)
Alborán Sea
Melilla (SPAIN)
Oran
Algiers
Tunis
Crete
Scale 1: 19,500,000
Lambert Conformal Conic Projection, standard parallels 40°N and 56°N
300 Kilometers
300 Miles
Valletta
MALTA
Rabat
Casablanca
MOROCCO
ALGERIA
TUNISIA
Boundary representation not necessarily authorit

Outline

Why multipartite entanglement is important?

- Many experiments are aiming to create entangled states with many atoms.
- Full tomography is not possible, we still have to say something meaningful.
- Only collective quantities can be measured.
- Thus, entanglement detection seems to be a good idea ...

Why is this challenging?

- It could happen that
  - it is not possible to create large scale entanglement in a system that is not completely isolated.
  - such entanglement is created, but we cannot verify its presence, since we can measure few things.

Outline

Entanglement

A state is (fully) separable if it can be written as

$$\sum_k p_k \varrho_1^{(k)} \otimes \varrho_2^{(k)} \otimes ... \otimes \varrho_N^{(k)}.$$

If a state is not separable then it is entangled.

Multipartite entanglement

- Three qubit pure states, 6 SLOCC classes.
  W. Dür, G. Vidal, J. I. Cirac, Phys. Rev. A 62, 062314 (2000).
- Three qubit mixed states, 6 classes.
  A Acín, D Bruss, M Lewenstein, A Sanpera, Phys. Rev. Lett. 87, 040401 (2001).
- Four qubits, infinite number of classes, 9 families.
  F. Verstraete, J. Dehaene, B. De Moor, H. Verschelde, Phys. Rev. A, 65: 052112 (2002).

k-producibility/k-entanglement

A pure state is k-producible if it can be written as

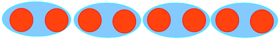
$$|\Phi\rangle = |\Phi_1\rangle \otimes |\Phi_2\rangle \otimes |\Phi_3\rangle \otimes |\Phi_4\rangle ....$$

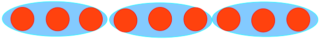
where $|\Phi_l\rangle$ are states of at most k qubits.

A mixed state is k-producible, if it is a mixture of k-producible pure states.

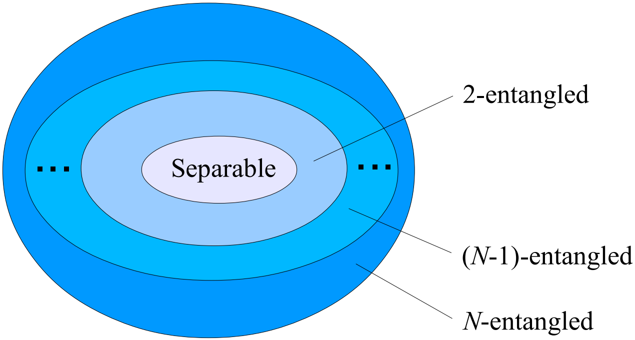
e.g., O. Gühne and GT, New J. Phys 2005.

- If a state is not k-producible, then it is at least $(k+1)$-particle entangled.





k-producibility/k-entanglement II



$(\|00\rangle + \|11\rangle) \otimes (\|00\rangle + \|11\rangle) \otimes (\|00\rangle + \|11\rangle)$	2-entangled
$(\|000\rangle + \|111\rangle) \otimes (\|000\rangle + \|111\rangle)$	3-entangled
$(\|0000\rangle + \|1111\rangle) \otimes (\|0\rangle + \|1\rangle)$	4-entangled

k-entanglement means real k-particle quantumness

- k-entanglement means that we could not make trivially the experiment from $(k-1)$-particle experiments.
- The state is not a mixture of product states

$$\varrho_1 \otimes \varrho_2 \otimes \varrho_3 \otimes \ldots$$

such that all ϱ_l has at most $(k-1)$ qubits.

Outline

Many-particle systems for j=1/2

- For spin-$\frac{1}{2}$ particles, we can measure the collective angular momentum operators:

$$J_l := \frac{1}{2} \sum_{k=1}^{N} \sigma_l^{(k)},$$

  where $l = x, y, z$ and $\sigma_l^{(k)}$ a Pauli spin matrices.

- We measure the expectation values $\langle J_l \rangle$.

- We can also measure the variances

$$(\Delta J_l)^2 := \langle J_l^2 \rangle - \langle J_l \rangle^2.$$

Outline

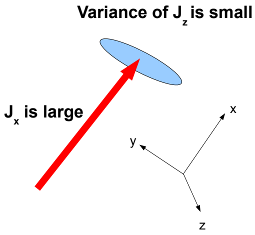
The standard spin-squeezing criterion

The spin squeezing criterion for entanglement detection is

$$\xi_s^2 = N \frac{(\Delta J_z)^2}{\langle J_x \rangle^2 + \langle J_y \rangle^2}.$$

[A. Sørensen, L.M. Duan, J.I. Cirac, P. Zoller, Nature 409, 63 (2001).]

- If $\xi_s^2 < 1$ then the state is entangled.
- States detected are like this:



- They are good for metrology!

Multipartite entanglement in spin squeezing

- We consider pure k-producible states of the form

$$|\Psi\rangle = \otimes_{l=1}^{M} |\psi_l\rangle,$$

where $|\psi_l\rangle$ is the state of at most k qubits.

Extreme spin squeezing

The spin-squeezing criterion for k-producible states is

$$(\Delta J_z)^2 \geqslant J_{\max} F_{\frac{k}{2}}\left(\frac{\sqrt{\langle J_x\rangle^2 + \langle J_y\rangle^2}}{J_{\max}}\right),$$

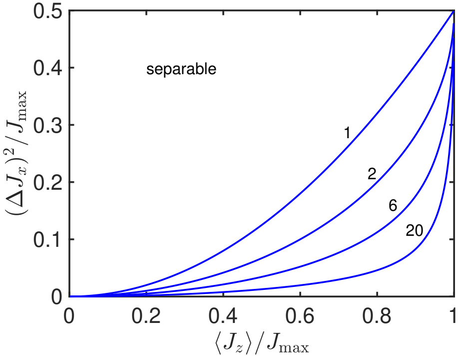
where $J_{\max} = \frac{N}{2}$ and we use the definition

$$F_j(X) := \frac{1}{j} \min_{\frac{\langle j_x\rangle}{j}=X} (\Delta j_z)^2.$$

Sørensen and Mølmer, Phys. Rev. Lett. 86, 4431 (2001); experimental test: Gross, Zibold, Nicklas, Esteve, Oberthaler, Nature 464, 1165 (2010).

Multipartite entanglement in spin squeezing II

- Larger and larger multipartite entanglement is needed to larger and larger squeezing ("extreme spin squeezing").



- $N = 100$ spin-1/2 particles, $J_{\max} = N/2$.

Sørensen and Mølmer, Phys. Rev. Lett. 86, 4431 (2001); experimental test: Gross, Zibold, Nicklas, Esteve, Oberthaler, Nature 464, 1165 (2010).

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Generalized spin squeezing criteria for $j=\frac{1}{2}$

- Let us assume that for a system we know only

$$\vec{J} := (\langle J_{\rm x}\rangle, \langle J_{\rm y}\rangle, \langle J_{\rm z}\rangle),$$
$$\vec{K} := (\langle J_{\rm x}^2\rangle, \langle J_{\rm y}^2\rangle, \langle J_{\rm z}^2\rangle).$$

- Then any state violating the following inequalities is entangled:

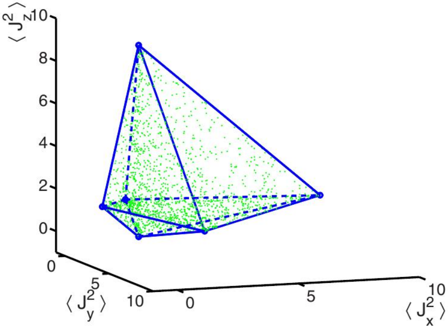
$$\langle J_x^2\rangle + \langle J_y^2\rangle + \langle J_z^2\rangle \le \frac{N(N+2)}{4},$$
$$(\Delta J_x)^2 + (\Delta J_y)^2 + (\Delta J_z)^2 \ge \frac{N}{2}, \qquad \text{(singlet states)}$$
$$\langle J_k^2\rangle + \langle J_l^2\rangle \le (N-1)(\Delta J_m)^2 + \frac{N}{2}, \qquad \text{(Dicke states)}$$
$$(N-1)\left[(\Delta J_k)^2 + (\Delta J_l)^2\right] \ge \langle J_m^2\rangle + \frac{N(N-2)}{4},$$

where k, l, m take all the possible permutations of x, y, z.

singlets: GT, Phys. Rev. A 69, 052327 (2004);
all Eqs.: GT, C. Knapp, O. Gühne, and H.J. Briegel, PRL 99, 250405 (2007);
spin-j: G. Vitagliano, P. Hyllus, I. L. Egusquiza, GT, PRL 107, 240502 (2011).

Generalized spin squeezing criteria for $j = \frac{1}{2}$ II

- Separable states are in the polytope



- We set $\langle J_l \rangle = 0$ for $l = x, y, z$.

Spin squeezing criteria – Two-particle correlations

All quantities depend only on two-particle correlations

$$\langle J_l \rangle = N \langle j_l \otimes \mathbb{1} \rangle_{\varrho_{2\mathrm{p}}}; \quad \langle J_l^2 \rangle = \frac{N}{4} + N(N-1) \langle j_l \otimes j_l \rangle_{\varrho_{2\mathrm{p}}}.$$

- Average 2-particle density matrix

$$\varrho_{2\mathrm{p}} = \frac{1}{N(N-1)} \sum_{n \neq m} \varrho_{mn}.$$

- We can detect states with a separable $\varrho_{2\mathrm{p}}$!

Singlet state

- Singlet states are ground states of antiferromagnetic Hamiltonians.

- The permutationally invariant singlet is

$$\varrho_{\text{singlet}} \propto \lim_{T\to 0} e^{-\frac{J_x^2+J_y^2+J_z^2}{T}}.$$

- For such a state, for large N we have

$$\varrho_{\text{2p}} \approx \frac{\mathbb{1}}{4},$$

  still it is detected as entangled by our criterion!

- Such a state has been created in cold atoms.

Singlet state experiments

- Creating singlets by squeezing the uncertainties of spin components $\rightarrow$ incoherent process, "creating entanglement with decoherence".

G. Tóth and M. W. Mitchell, Generation of macroscopic singlet states in atomic ensembles, New J. Phys. 12 053007 (2010).

- Entanglement in singlet states in $> 10^6$ cold Rb atoms.

N. Behbood, F. Martin Ciurana, G. Colangelo, M. Napolitano, GT, R. J. Sewell, and M. W. Mitchell, Generation of Macroscopic Singlet States in a Cold Atomic Ensemble, Phys. Rev. Lett. 113, 093601 (2014).

- Entanglement in singlet states in $> 10^{13}$ hot alkali atoms.

J. Kong, R. Jiménez-Martínez, C. Troullinou, V. G. Lucivero, GT, and Morgan W. Mitchell, Measurement-induced, spatially-extended entanglement in a hot, strongly-interacting atomic system,
Nat. Commun. 11, 2415 (2020).

Applications of singlets

- Singlets are invariant under homogeneous fields.
- Gradient field destroys the singlet $\rightarrow$ Gradient metrology.

I. Apellaniz, I. Urizar-Lanz, Z. Zimborás, P. Hyllus, and GT, Precision bounds for gradient magnetometry with atomic ensembles, Phys. Rev. A 97, 053603 (2018).

S. Altenburg, M. Oszmaniec, S. Wölk, O. Gühne, Estimation of gradients in quantum metrology, Phys. Rev. A 96, 042319 (2017).

I. Urizar-Lanz, P. Hyllus, I. L. Egusquiza, M. W. Mitchell, and GT, Macroscopic singlet states for gradient magnetometry, Phys. Rev. A 88, 013626 (2013).

Entanglement conditions based on the two-body density matrix

- Spin squeezing conditions with collective variables based on the two-body density matrix:

Spin Squeezing Inequalities and Entanglement of *N* Qubit States, J. K. Korbicz, J. I. Cirac, and M. Lewenstein, Phys. Rev. Lett. 95, 120502 (2005).

Outline

1. **Motivation**
   - Why entanglement is important?
2. **Spin squeezing and entanglement**
   - Entanglement
   - Collective measurements
   - The original spin-squeezing criterion
   - Generalized criteria for $j = \frac{1}{2}$, singlet states
3. **Detecting multipartite entanglement of Dicke states**
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Generalized spin squeezing criteria for $j = \frac{1}{2}$

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GT, C. Knapp, O. Gühne, and H.J. Briegel, PRL 99, 250405 (2007);
spin-j: G. Vitagliano, P. Hyllus, I. L. Egusquiza, GT, PRL 107, 240502 (2011).

Dicke states

- Dicke states: eigenstates of $\vec{J}^2 = J_x^2 + J_y^2 + J_z^2$ and J_z.
- Symmetric Dicke states of spin-1/2 particles, with $\langle J_z \rangle = \langle J_z^2 \rangle = 0$

$$|D_N\rangle = \binom{N}{\frac{N}{2}}^{-\frac{1}{2}} \sum_k \mathcal{P}_k \left(|0\rangle^{\otimes \frac{N}{2}} \otimes |1\rangle^{\otimes \frac{N}{2}} \right).$$

  Summing over all permutations.
  Due to symmetry, $\left\langle \vec{J}^2 \right\rangle$ is maximal.

- E.g., for four qubits they look like

$$|D_4\rangle = \frac{1}{\sqrt{6}} \left(|0011\rangle + |0101\rangle + |1001\rangle + |0110\rangle + |1010\rangle + |1100\rangle \right).$$

photons: N. Kiesel, C. Schmid, GT, E. Solano, H. Weinfurter, PRL 2007; Prevedel. *et al.,* PRL 2009; W. Wieczorek, R. Krischek, N. Kiesel, P. Michelberger, GT, H. Weinfurter, PRL 2009.

cold atoms: Lücke, Science 2011; Hamley *et al*, Nat. Phys. 2012.

Dicke states are useful because they ...

- ... possess strong multipartite entanglement, like GHZ states.

GT, JOSAB 2007.

- ... are optimal for quantum metrology, similarly to GHZ states.

Hyllus *et al.*, PRA 2012; Lücke *et al.*, Science 2011;
GT, PRA 2012;
GT and Apellaniz, J. Phys. A, special issue for “50 year of Bell’s theorem”, 2014.

- ... are macroscopically entangled, like GHZ states.

Fröwis, Dür, PRL 2011.

Spin Squeezing Inequality for Dicke states

- Let us rewrite the third inequality. For separable states

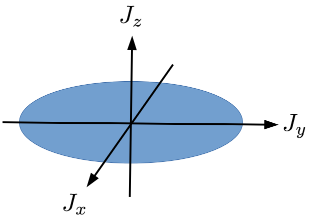
$$\langle J_x^2\rangle + \langle J_y^2\rangle - \frac{N}{2} \leq (N-1)(\Delta J_z)^2$$

  holds.

- It detects states close to Dicke states since

$$\langle J_x^2 + J_y^2\rangle = \frac{N}{2}\left(\frac{N}{2}+1\right) = \max.,$$
$$\langle J_z^2\rangle = 0.$$

- "Pancake" like uncertainty ellipse.



Multipartite entanglement around Dicke states

- Measure the same quantities as before

$$(\Delta J_z)^2$$

and

$$\langle J_x^2 + J_y^2 \rangle.$$

- In contrast, for the original spin-squeezing criterion we measured $(\Delta J_z)^2$ and $\langle J_x \rangle^2 + \langle J_y \rangle^2$.
- Pioneering work: linear condition of Luming Duan, Phys. Rev. Lett. (2011). See also Zhang, Duan, New. J. Phys. (2014).

Multipartite entanglement - Our condition

- Sørensen-Mølmer condition for k-producible states

$$(\Delta J_z)^2 \geqslant J_{\max} F_{\frac{k}{2}} \left(\frac{\sqrt{\langle J_x \rangle^2 + \langle J_y \rangle^2}}{J_{\max}} \right).$$

- Combine it with

$$\langle J_x^2 + J_y^2 \rangle \leqslant J_{\max}(\tfrac{k}{2} + 1) + \langle J_x \rangle^2 + \langle J_y \rangle^2,$$

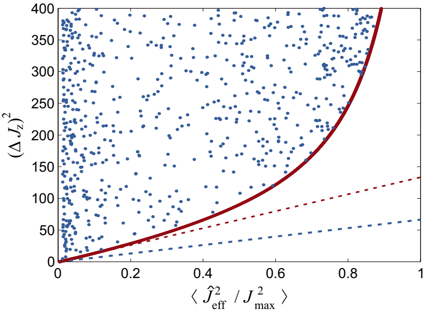
which is true for pure k-producible states. (Remember, $J_{\max} = \frac{N}{2}$.)

Condition for entanglement detection around Dicke states

$$(\Delta J_z)^2 \geqslant J_{\max} F_{\frac{k}{2}} \left(\frac{\sqrt{\langle J_x^2 + J_y^2 \rangle - J_{\max}(\frac{k}{2} + 1)}}{J_{\max}} \right).$$

Due to convexity properties of the expression, this is also valid to mixed separable states.

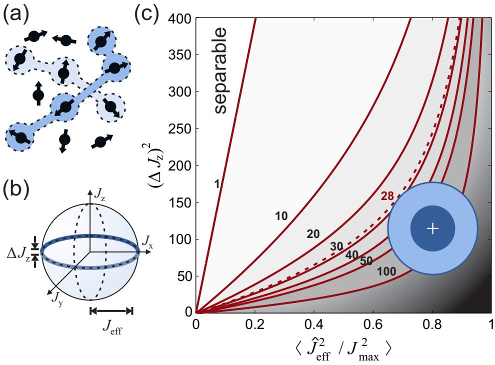
Concrete example



- $N = 8000$ particles, and $J_{\text{eff}} = J_x^2 + J_y^2$.
- Red curve: boundary for 28-particle entanglement.
- Blue dashed line: linear condition given in L.-M. Duan, Phys. Rev. Lett. 107, 180502 (2011).
- Red dashed line: tangent of our curve.

Multipartite entanglement

- Bose-Einstein condensate, 8000 particles. 28-particle entanglement is detected.



$$J^2_{\mathrm{eff}} = J_x^2 + J_y^2, \quad J_{\max} = \frac{N}{2}.$$

B. Lücke, J. Peise, G. Vitagliano, J. Arlt, L. Santos, GT, and C. Klempt, PRL 112, 155304 (2014).

Multipartite entanglement

- More than 600-particle entanglement in a Dicke state of 11000 atoms.

Li. Yi-Quan Zou *et al.*, PNAS 115 (25) 6381-6385 (2018).

Spin squeezing criteria – Two-particle correlations

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- Average 2-particle density matrix

$$\varrho_{2\mathrm{p}} = \frac{1}{N(N-1)} \sum_{n \neq m} \varrho_{mn}.$$

- We can even detect multipartite entanglement knowing only two-body correlations!

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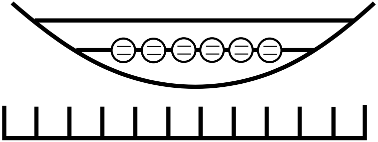
Bipartite entanglement from bosonic multipartite entanglement

- In the BEC, "all the particles are at the same place."
- In the usual formulation, entanglement is between spatially separated parties.
- Is multipartite entanglement within a BEC useful/real?
- Answer: yes!

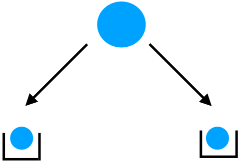
Bipartite entanglement from bosonic multipartite entanglement II

- Dilute cloud argument



See, e.g., P. Hyllus, L. Pezzé, A Smerzi and GT, PRA 86, 012337 (2012)

$|n_0 = 1\rangle |n_1 = 1\rangle$

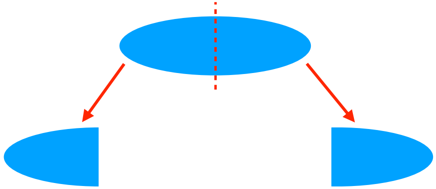


$\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$

Bipartite entanglement from bosonic multipartite entanglement III

- After splitting it into two, we have bipartite entanglement if we had before multipartite entanglement.

- The splitting does not generate entanglement, if we consider projecting to a fixed particle number.



N. Killoran, M. Cramer, and M. B. Plenio, PRL 112, 150501 (2014).

Outline

Experiment in the group of Carsten Klempt at the University of Hannover

- Rubidium BEC, spin-1 atoms.

- Initially all atoms in the spin state $|j_z = 0\rangle$.

- Dynamics
$$H = a_0^2 a_{+1}^\dagger a_{-1}^\dagger + (a_0^\dagger)^2 a_{+1} a_{-1}.$$
Tunneling from mode 0 to the mode $+1$ and -1.

- Understanding the tunneling process
$$|j_z = 0\rangle|j_z = 0\rangle \rightarrow \frac{1}{\sqrt{2}}(|j_z = +1\rangle|j_z = -1\rangle + |j_z = -1\rangle|j_z = +1\rangle)$$
$$= \text{Dicke state of 2 particles.}$$

Experiment in the group of Carsten Klempt at the University of Hannover II

- After some time, we have a state

$$|n_0, n_{-1}, n_{+1}\rangle = |N-2n, n, n\rangle.$$

- That is, $N-2n$ particles remained in the $|j_z = 0\rangle$ state, while $2n$ particles form a symmetric Dicke state given as

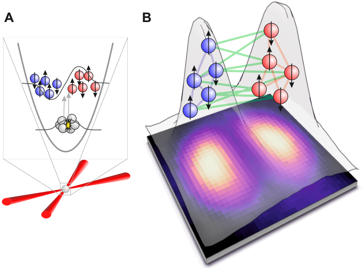
$$|D_N\rangle = \binom{N}{\frac{N}{2}}^{-\frac{1}{2}} \sum_k \mathcal{P}_k \left(|0\rangle^{\otimes \frac{N}{2}} \otimes |1\rangle^{\otimes \frac{N}{2}} \right),$$

where we use $|0\rangle$ and $|1\rangle$ instead of $|j_z = -1\rangle$ and $|j_z = +1\rangle$.

- Half of the atoms in state $|0\rangle$, half of the atoms in state $|1\rangle$ + symmerization.

Experiment in the group of Carsten Klempt at the University of Hannover III

- Important: first excited spatial mode of the trap was used, not the ground state mode.
- It has two "bumps" rather than one, hence they had a split Dicke state.



K. Lange, J. Peise, B. Lücke, I. Kruse, G. Vitagliano, I. Apellaniz, M. Kleinmann, GT, and C. Klempt, Entanglement between two spatially separated atomic modes, Science 360, 416 (2018).

Correlations for Dicke states

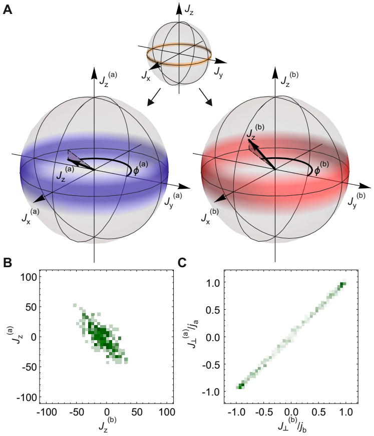
- For the Dicke state

$$(\Delta(J_x^a - J_x^b))^2 \approx 0,$$
$$(\Delta(J_y^a - J_y^b))^2 \approx 0,$$
$$(\Delta J_z)^2 = 0.$$

- Measurement results on well "*b*" can be predicted from measurements on "*a*"

$$\begin{aligned} J_x^b &\approx J_x^a, \\ J_y^b &\approx J_y^a, \\ J_z^b &= -J_z^a. \end{aligned}$$

Correlations for Dicke states - experimental results



Here, $J_\perp^{(n)} = \cos\alpha J_x^{(n)} + \sin\alpha J_y^{(n)}$.

Experiment in K. Lange *et al.*, Science 334, 773–776 (2011).

Outline

Normalized variables

- Let us introduce the normalized variables

$$\mathcal{J}_m^n = \frac{J_m^n}{\sqrt{j_n(j_n+1)}} \approx \frac{J_m^n}{N_n},$$

where $m = x, y$ and $n = a, b$ (i.e., left well, right well), the total spin is

$$j_n = \frac{N_n}{2},$$

- Normalized variables → resistance to experimental imperfections.

The two-well entanglement criterion

Suggestion of the experimentalists: we need a product criterion, since it is good for realistic noise.

Main result I

For separable states,

$$\left[(\Delta J_z)^2 + \frac{1}{4}\right]\left[(\Delta \mathcal{J}_x^-)^2 + (\Delta \mathcal{J}_y^-)^2\right] \geq \frac{1}{16}\left\langle \mathcal{J}_x^2 + \mathcal{J}_y^2 \right\rangle^2$$

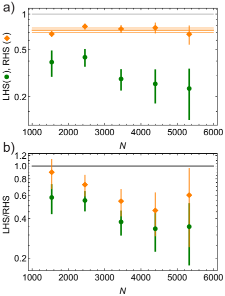
holds.

Here,

$$\begin{aligned} J_z &= J_z^a + J_z^b, \\ \mathcal{J}_m^- &= \mathcal{J}_m^a - \mathcal{J}_m^b \end{aligned}$$

for $m = x, y$.

Violation of the criterion: entanglement is detected II



LHS/RHS (top) for Quantum 2023, and (bottom) for Science 2018.

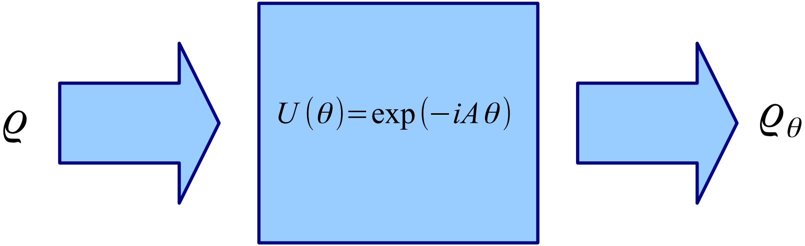
K. Lange, J. Peise, B. Lücke, I. Kruse, G. Vitagliano, I. Apellaniz, M. Kleinmann, GT, C. Klempt, Entanglement between two spatially separated atomic modes, Science 360, 416 (2018).

G. Vitagliano, I. Apellaniz, M. Fadel, M. Kleinmann, B. Lücke, C. Klempt, GT, Number-phase uncertainty relations and bipartite entanglement detection in spin ensembles, Quantum 7, 914 (2023).

Outline

Quantum metrology

- Fundamental task in metrology



- We have to estimate θ in the dynamics

$$U = \exp(-iA\theta).$$

The quantum Fisher information

- Cramér-Rao bound on the precision of parameter estimation

$$(\Delta\theta)^2 \geq \frac{1}{\nu F_Q[\varrho, A]},$$

  where $F_Q[\varrho, A]$ is the quantum Fisher information, and ν is the number of independent repetitions.

- The quantum Fisher information is

$$F_Q[\varrho, A] = 2 \sum_{k,l} \frac{(\lambda_k - \lambda_l)^2}{\lambda_k + \lambda_l} |\langle k|A|l\rangle|^2,$$

  where $\varrho = \sum_k \lambda_k |k\rangle\langle k|$.

Linear interferometers

- We consider
$$F_Q[\varrho, J_l]$$
for $l = x, y, z$.

- The Hamiltonian is J_l, which does not contain interactions.

The quantum Fisher information vs. entanglement

- For separable states of N spin-1/2 particles (qubits)

$$F_Q[\varrho, J_l] \leq N, \qquad l = x, y, z.$$

Pezze, Smerzi, Phys. Rev. Lett. 102, 100401 (2009);
Hyllus, Gühne, Smerzi, Phys. Rev. A 82, 012337 (2010).

- For states with at most k-qubit entanglement (tight bound if k is a divisor of N)

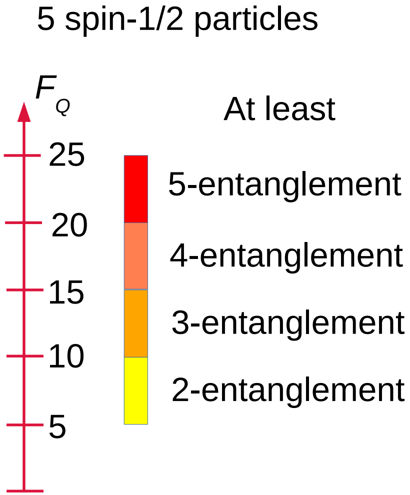
$$F_Q[\varrho, J_l] \leq kN.$$

P. Hyllus *et al.* Phys. Rev. A 85, 022321 (2012);
GT, Phys. Rev. A 85, 022322 (2012).

- Bound for all quantum states of N qubits

$$F_Q[\varrho, J_l] \leq N^2.$$

The quantum Fisher information vs. entanglement



(For simplicity, we used $F_Q[\varrho, J_l] \leq kN$, which is not tight.)

Let us use the Cramér-Rao bound

- For separable states

$$(\Delta\theta)^2 \geq \frac{1}{\nu N}, \qquad l = x, y, z.$$

Pezze, Smerzi, Phys. Rev. Lett. 102, 100401 (2009);
Hyllus, Gühne, Smerzi, Phys. Rev. A 82, 012337 (2010).

- For states with at most k-particle entanglement (tight bound if k is a divisor of N)

$$(\Delta\theta)^2 \geq \frac{1}{\nu k N}.$$

P. Hyllus *et al.* Phys. Rev. A 85, 022321 (2012);
GT, Phys. Rev. A 85, 022322 (2012).

- Bound for all quantum states

$$(\Delta\theta)^2 \geq \frac{1}{\nu N^2}.$$

Outline

Metrology with Dicke states

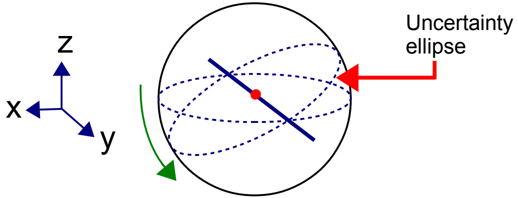
- For Dicke state

$$\langle J_l \rangle = 0, l = x, y, z, \quad \langle J_z^2 \rangle = 0, \quad \langle J_x^2 \rangle = \langle J_y^2 \rangle = \text{large}.$$

- Linear metrology

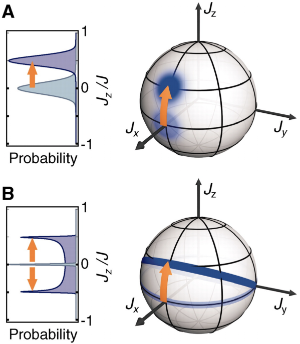
$$U = \exp(-iJ_y\theta).$$

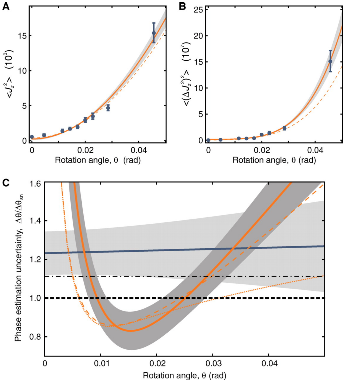
- Measure $\langle J_z^2 \rangle$ to estimate θ. (We cannot measure first moments, since they are zero.)



Metrology with Dicke states

Experiment with cold gas of 8000 atoms.





Lücke M. Scherer, Kruse, Pezzé, Deuretzbacher, Hyllus, Topic, Peise, Ertmer, Arlt, Santos, Smerzi, Klempt, Science 2011.

Multipartite entanglement vs. metrology

- Conclusion: Multipartite entanglement is needed for high precision quantum metrology.
- Without high level of multipartite entanglement, we cannot have high precision quantum metrology.

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- Theory: O. Gühne, Siegen U., Germany
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- Z. Zimborás, Wigner, Hungary
- H.-J. Briegel, Insbruck, Austria
- L. Santos, Hannover U., Germany
- L. Pezze, A. Smerzi, LENS, Firenze
- Photonic experiments: H. Weinfurter, N. Kiesel, W. Wieczorek, L. Knips, MPQ München
- Dicke state experiments with BEC: C. Klempt, I. Kruse, J. Peise, K. Lange, B. Lücke, Hannover U., Germany; J. Arlt, Arhus U., Denmark
- Singlet state experiments with cold atoms: M. W. Mitchell, R. J. Sewell, N. Behbood, F. Martin Ciurana, G. Colangelo, M. Napolitano, J. Kong, R. Jiménez-Martínez, C. Troullinou, V. G. Lucivero, ICFO, Barcelona

Summary

- We discussed entanglement detection in particle ensembles.

THANK YOU FOR YOUR ATTENTION!

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