

Detection of multipartite entanglement close to symmetric Dicke states

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1 Motivation

- Why multipartite entanglement is important?

2 Spin squeezing and entanglement

- Entanglement
- Collective measurements
- The original spin-squeezing criterion
- Generalized criteria for $j = \frac{1}{2}$

3 Spin squeezing for Dicke states

- Entanglement detection close to Dicke states
- Detection of multipartite entanglement close to Dicke states
- Our conditions are stronger than the original conditions

Why multipartite entanglement is important?

- Full tomography is not possible, we still have to say something meaningful.
- Claiming “entanglement” is not sufficient for many particles.
- Many experiments are aiming to create entangled states with many atoms.
- Only collective quantities can be measured.

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Entanglement

A state is **(fully) separable** if it can be written as

$$\sum_k p_k \varrho_1^{(k)} \otimes \varrho_2^{(k)} \otimes \dots \otimes \varrho_N^{(k)}.$$

If a state is not separable then it is **entangled**.

A pure state is k -producible if it can be written as

$$|\Phi\rangle = |\Phi_1\rangle \otimes |\Phi_2\rangle \otimes |\Phi_3\rangle \otimes |\Phi_4\rangle \dots$$

where $|\Phi_j\rangle$ are states of at most k qubits.

A mixed state is k -producible, if it is a mixture of k -producible pure states.

[e.g., O. Gühne and GT, New J. Phys 2005.]

- If a state is not k -producible, then it is at least $(k + 1)$ -particle entangled.

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Many-particle systems for $j=1/2$

- For spin- $\frac{1}{2}$ particles, we can measure the **collective angular momentum operators**:

$$J_l := \frac{1}{2} \sum_{k=1}^N \sigma_l^{(k)},$$

where $l = x, y, z$ and $\sigma_l^{(k)}$ a Pauli spin matrices.

- We can also measure the **variances**

$$(\Delta J_l)^2 := \langle J_l^2 \rangle - \langle J_l \rangle^2.$$

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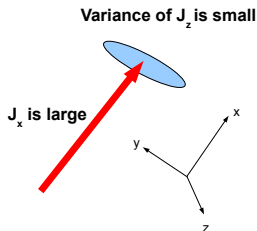
The standard spin-squeezing criterion

The **spin squeezing criteria for entanglement detection** is

$$\xi_s^2 = N \frac{(\Delta J_z)^2}{\langle J_x \rangle^2 + \langle J_y \rangle^2}.$$

[A. Sørensen, L.M. Duan, J.I. Cirac, P. Zoller, Nature 409, 63 (2001).]

- If $\xi_s^2 < 1$ then the state is entangled.
- States detected are like this:



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Generalized spin squeezing criteria for $j = \frac{1}{2}$

- Let us assume that for a system we know only

$$\vec{J} := (\langle J_x \rangle, \langle J_y \rangle, \langle J_z \rangle),$$

$$\vec{K} := (\langle J_x^2 \rangle, \langle J_y^2 \rangle, \langle J_z^2 \rangle).$$

- Then any state violating the following inequalities is entangled:

$$\langle J_x^2 \rangle + \langle J_y^2 \rangle + \langle J_z^2 \rangle \leq \frac{N(N+2)}{4},$$

$$(\Delta J_x)^2 + (\Delta J_y)^2 + (\Delta J_z)^2 \geq \frac{N}{2},$$

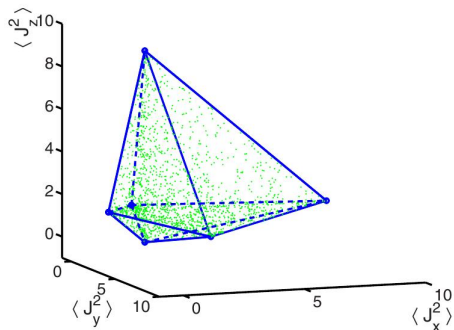
$$\langle J_k^2 \rangle + \langle J_l^2 \rangle \leq (N-1)(\Delta J_m)^2 + \frac{N}{2},$$

$$(N-1) [(\Delta J_k)^2 + (\Delta J_l)^2] \geq \langle J_m^2 \rangle + \frac{N(N-2)}{4},$$

where k, l, m take all the possible permutations of x, y, z .

Generalized spin squeezing criteria for $j = \frac{1}{2} \mathbb{I}$

- Separable states are in the polytope



- We set $\langle J_l \rangle = 0$ for $l = x, y, z$.

Spin squeezing criteria – Two-particle correlations

All quantities needed can be obtained with two-particle correlations

$$\langle J_I \rangle = N \langle j_I \otimes \mathbb{1} \rangle_{\rho_{2p}}; \quad \langle J_I^2 \rangle = \frac{N}{4} + N(N-1) \langle j_I \otimes j_I \rangle_{\rho_{2p}}.$$

- Here, the average 2-particle density matrix is defined as

$$\rho_{2p} = \frac{1}{N(N-1)} \sum_{n \neq m} \rho_{mn}.$$

- Still, we can detect states with a separable ρ_{2p} .
- Still, as we will see, we can even detect multipartite entanglement!

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Dicke states

- Symmetric Dicke states with $\langle J_z \rangle = 0$ (simply “Dicke states” in the following) are defined as

$$|D_N\rangle = \binom{N}{\frac{N}{2}}^{-\frac{1}{2}} \sum_k \mathcal{P}_k \left(|0\rangle^{\otimes \frac{N}{2}} \otimes |1\rangle^{\otimes \frac{N}{2}} \right).$$

- E.g., for four qubits they look like

$$|D_4\rangle = \frac{1}{\sqrt{6}} (|0011\rangle + |0101\rangle + |1001\rangle + |0110\rangle + |1010\rangle + |1100\rangle).$$

[photons: Kiesel *et al.*, Phys. Rev. Lett. 2007; Prevedel *et al.*, Phys. Rev. Lett 2007; Wieczorek *et al.*, Phys. Rev. Lett. 2009]

[cold atoms: Lücke *et al.*, Science 2011; Hamley *et al.*, Science 2011]

Dicke states are useful because they ...

- ... possess strong multipartite entanglement, like GHZ states.

[GT, JOSAB 2007.]

- ... are optimal for quantum metrology, similarly to GHZ states.

[Hyllus *et al.*, PRA 2012; Lücke *et al.*, Science 2011.]

[GT, PRA 2012;

GT and Apellaniz, J. Phys. A, special issue for “50 year of Bell’s theorem”, 2014.]

- ... are macroscopically entangled.

[Fröwis, Dür, PRL 2011]

Spin Squeezing Inequality for Dicke states

- Let us rewrite the third inequality

$$\langle J_k^2 \rangle + \langle J_l^2 \rangle - \frac{N}{2} \leq (N-1)(\Delta J_m)^2.$$

- It detects states close to Dicke states since

$$\langle J_x^2 + J_y^2 \rangle = \frac{N}{2} \left(\frac{N}{2} + 1 \right) = \max.,$$
$$\langle J_z^2 \rangle = 0.$$

Spin Squeezing Inequality for Dicke states II

Based on the above inequality, we define a **new spin squeezing parameter**

$$\xi_{\text{os}}^2 = \frac{RHS}{LHS} = (N-1) \frac{(\Delta J_z)^2}{\langle J_x^2 + J_y^2 \rangle - \frac{N}{2}}$$

[Vitagliano, Apellaniz, Egusquiza, GT, PRA (2014)]

- For our Dicke state, the numerator is minimal, the denominator is maximal, $\xi_{\text{os}}^2 = 0$.
- The original spin squeezing parameter would not detect the Dicke states, since

$$\xi_s^2 = N \frac{(\Delta J_z)^2}{\langle J_x \rangle^2 + \langle J_y \rangle^2} = N \frac{(\Delta J_z)^2}{0} = \infty.$$

Fully polarized states

- Relation between the second moments and the expectation value

$$\langle J_x^2 \rangle = \langle J_x \rangle^2 + (\Delta J_x)^2 \geq \langle J_x \rangle^2.$$

- For states polarized in the x-direction and spin squeezed along the z-direction, for $N \gg 1$, we have

$$\langle J_x^2 \rangle \approx \langle J_x \rangle^2 \gg N.$$

Hence, for fully polarized states

$$\xi_{\text{os}}^2 = (N-1) \frac{(\Delta J_z)^2}{\langle J_x^2 + J_y^2 \rangle - \frac{N}{2}} \approx \xi_s^2 = N \frac{(\Delta J_z)^2}{\langle J_x \rangle^2 + \langle J_y \rangle^2}.$$

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Multipartite entanglement in spin squeezing

- We consider pure k -producible states of the form

$$|\Psi\rangle = \otimes_{n=1}^M |\psi^{(n)}\rangle,$$

where $|\psi^{(n)}\rangle$ is the state of at most k qubits.

The **spin-squeezing criterion for k -producible states** is

$$(\Delta J_z)^2 \geq J_{\max} F_{\frac{k}{2}} \left(\frac{\sqrt{\langle J_x \rangle^2 + \langle J_y \rangle^2}}{J_{\max}} \right),$$

where $J_{\max} = \frac{N}{2}$ and we use the definition

$$F_j(X) := \frac{1}{j} \min_{\frac{\langle j_x \rangle}{j} = X} (\Delta j_z)^2.$$

[Sørensen and Mølmer, Phys. Rev. Lett. 86, 4431 (2001);
experimental test: C. Gross *et al.*, Nature 464, 1165 (2010).]

Multipartite entanglement around Dicke states

- Measure the same quantities as before

$$(\Delta J_z)^2$$

and

$$\langle J_x^2 + J_y^2 \rangle.$$

- In contrast, for the original spin-squeezing criterion we measured $(\Delta J_z)^2$ and $\langle J_x \rangle^2 + \langle J_y \rangle^2$.
- Pioneering work: linear condition of Luming Duan, Phys. Rev. Lett. (2011). See also Zhang, Duan, arxiv (2014).

Multipartite entanglement - Our condition

- Sørensen-Mølmer condition for k -producible states

$$(\Delta J_z)^2 \geq J_{\max} F_{\frac{k}{2}} \left(\frac{\sqrt{\langle J_x \rangle^2 + \langle J_y \rangle^2}}{J_{\max}} \right).$$

- Combine it with

$$\langle J_x^2 + J_y^2 \rangle \leq J_{\max} \left(\frac{k}{2} + 1 \right) + \langle J_x \rangle^2 + \langle J_y \rangle^2,$$

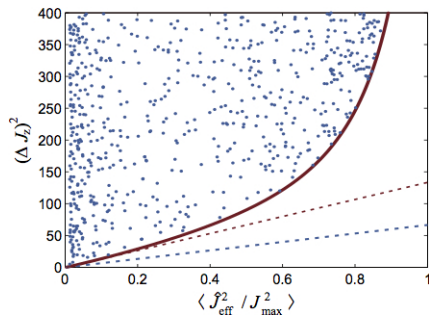
which is true for pure k -producible states. (Remember, $J_{\max} = \frac{N}{2}$.)

Condition for **entanglement detection around Dicke states**

$$(\Delta J_z)^2 \geq J_{\max} F_{\frac{k}{2}} \left(\frac{\sqrt{\langle J_x^2 + J_y^2 \rangle - J_{\max} \left(\frac{k}{2} + 1 \right)}}{J_{\max}} \right).$$

Due to convexity properties of the expression, this is also valid to mixed separable states.

Concrete example



- $N = 8000$ particles, and $J_{\text{eff}} = J_x^2 + J_y^2$.
- **Red curve:** boundary for 28-particle entanglement.
- **Blue dashed line:** linear condition given in [L.-M. Duan, Phys. Rev. Lett. 107, 180502 (2011).]
- **Red dashed line:** tangent of our curve.

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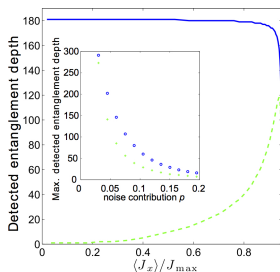
Our condition is stronger

- Consider spin squeezed states as ground states of

$$H(\Lambda) = J_Z^2 - \Lambda J_X.$$

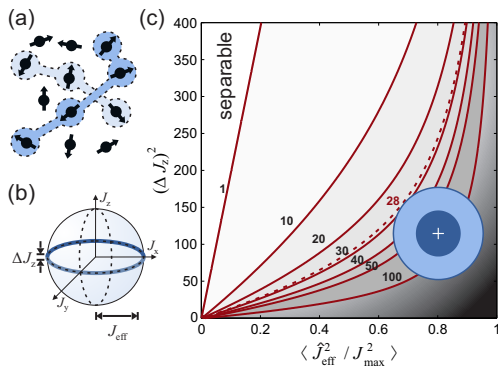
For $\Lambda = \infty$, the ground state is fully polarized. For $\Lambda = 0$, it is the symmetric Dicke state.

- Our condition vs. original condition for $N=4000$ and $p=0.05$



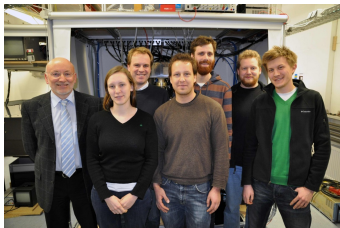
Experimental results

- Bose-Einstein condensate, 8000 particles. 28-particle entanglement is detected.



[Lücke *et al.*, Phys. Rev. Lett. 112, 155304 (2014).]

Project participants



C. Klempt, B. Lücke, J. Mahnke
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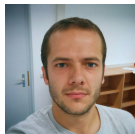


L. Santos

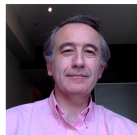
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Summary

- Detection of multipartite entanglement close to Dicke states, by measuring collective quantities only.
- The condition detects all entangled states that can be detected based on the measured quantities (i.e., it is optimal).

Vitagliano, Apellaniz, Egusquiza, GT, PRA (2014).

Lücke, Peise, Vitagliano, Arlt, Santos, GT, Klempt,
PRL 112, 155304 (2014)

(synopsis at physics.aps.org, Revista Española de Física).

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