Detecting metrologically useful multiparticle entanglement with few measurements: recent results

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Why multipartite entanglement is important?

- Full tomography is not possible, we still have to say something meaningful.
- Claiming “entanglement” is not sufficient for many particles.
- Many experiments are aiming to create entangled states with many atoms.
- Only collective quantities can be measured.

Outline

1. Introduction and motivation
2. **Spin squeezing and entanglement**
   - **Entanglement**
   - Collective measurements
   - The original spin-squeezing criterion
   - Generalized criteria for $j = \frac{1}{2}$
3. Spin squeezing for Dicke states
   - Entanglement detection close to Dicke states
   - Detection of multipartite entanglement close to Dicke states
4. Detecting metrologically useful entanglement
   - Basics of quantum metrology
   - Metrology with measuring $\langle J_z^2 \rangle$
   - Metrology with measuring any operator

Entanglement

A state is (fully) separable if it can be written as

$$\sum_k p_k \varrho_k^{(1)} \otimes \varrho_k^{(2)} \otimes ... \otimes \varrho_k^{(N)}.$$

If a state is not separable then it is entangled (Werner, 1989).

- Separable states remain separable under local operations. ("Local operations and classical communication")
- Separable states can be cerated without real quantum interaction. They are the "boring" states.

k-producibility/k-entanglement

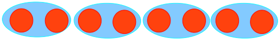
A pure state is k-producible if it can be written as

$$|\Phi\rangle = |\Phi_1\rangle \otimes |\Phi_2\rangle \otimes |\Phi_3\rangle \otimes |\Phi_4\rangle ....$$

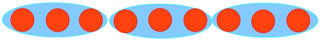
where $|\Phi_l\rangle$ are states of at most k qubits.

A mixed state is k-producible, if it is a mixture of k-producible pure states.
[e.g., O. Gühne and GT, New J. Phys 2005.]

- If a state is not k-producible, then it is at least $(k+1)$-particle entangled.



two-producible



three-producible

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Many-particle systems for j=1/2

- For spin-$\frac{1}{2}$ particles, we can measure the collective angular momentum operators:

$$J_l := \frac{1}{2} \sum_{k=1}^{N} \sigma_l^{(k)},$$

where $l = x, y, z$ and $\sigma_l^{(k)}$ a Pauli spin matrices.

- We can also measure the variances

$$(\Delta J_l)^2 := \langle J_l^2 \rangle - \langle J_l \rangle^2.$$

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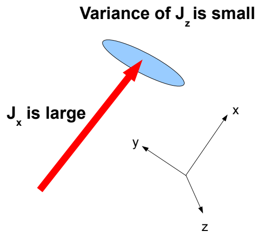
The standard spin-squeezing criterion

The spin squeezing criteria for entanglement detection is

$$\xi_s^2 = N \frac{(\Delta J_z)^2}{\langle J_x \rangle^2 + \langle J_y \rangle^2}.$$

[A. Sørensen, L.M. Duan, J.I. Cirac, P. Zoller, Nature 409, 63 (2001).]

- If $\xi_s^2 < 1$ then the state is entangled.
- States detected are like this:



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Generalized spin squeezing criteria for $j = \frac{1}{2}$

- Let us assume that for a system we know only

$$\vec{J} := (\langle J_x \rangle, \langle J_y \rangle, \langle J_z \rangle),$$
$$\vec{K} := (\langle J_x^2 \rangle, \langle J_y^2 \rangle, \langle J_z^2 \rangle).$$

- Then any state violating the following inequalities is entangled:

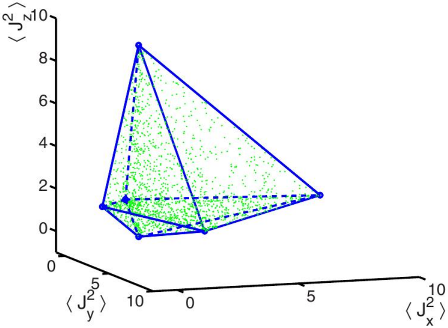
$$\langle J_x^2 \rangle + \langle J_y^2 \rangle + \langle J_z^2 \rangle \leq \frac{N(N+2)}{4},$$
$$(\Delta J_x)^2 + (\Delta J_y)^2 + (\Delta J_z)^2 \geq \frac{N}{2}, \qquad \text{(singlet)}$$
$$\langle J_k^2 \rangle + \langle J_l^2 \rangle \leq (N-1)(\Delta J_m)^2 + \frac{N}{2}, \qquad \text{(Dicke state)}$$
$$(N-1)\left[(\Delta J_k)^2 + (\Delta J_l)^2\right] \geq \langle J_m^2 \rangle + \frac{N(N-2)}{4},$$

where k, l, m take all the possible permutations of x, y, z.

[GT, C. Knapp, O. Gühne, and H.J. Briegel, PRL 99, 250405 (2007)]
[Singlets: Behbood *et al.*, Phys. Rev. Lett. 2014; GT, Mitchell, New. J. Phys. 2010.]

Generalized spin squeezing criteria for $j = \frac{1}{2}$ II

- Separable states are in the polytope



- We set $\langle J_l \rangle = 0$ for $l = x, y, z$.

Spin squeezing criteria – Two-particle correlations

All quantities needed can be obtained with two-particle correlations

$$\langle J_l \rangle = N \langle j_l \otimes \mathbb{1} \rangle_{\varrho_{2p}}; \quad \langle J_l^2 \rangle = \frac{N}{4} + N(N-1) \langle j_l \otimes j_l \rangle_{\varrho_{2p}}.$$

- Here, the average 2-particle density matrix is defined as

$$\varrho_{2p} = \frac{1}{N(N-1)} \sum_{n \neq m} \varrho_{mn}.$$

- Still, we can detect states with a separable ϱ_{2p}.
- Still, as we will see, we can even detect multipartite entanglement!

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Dicke states

- Symmetric Dicke states with $\langle J_z \rangle = 0$ (simply "Dicke states" in the following) are defined as

$$|D_N\rangle = \binom{N}{\frac{N}{2}}^{-\frac{1}{2}} \sum_k \mathcal{P}_k \left(|0\rangle^{\otimes \frac{N}{2}} \otimes |1\rangle^{\otimes \frac{N}{2}} \right).$$

- E.g., for four qubits they look like

$$|D_4\rangle = \frac{1}{\sqrt{6}} \left(|0011\rangle + |0101\rangle + |1001\rangle + |0110\rangle + |1010\rangle + |1100\rangle\right).$$

[photons: Kiesel, Schmid, GT, Solano, Weinfurter, PRL 2007;
Prevedel, Cronenberg, Tame, Paternostro, Walther, Kim, Zeilinger, PRL 2007;
Wieczorek, Krischek, Kiesel, Michelberger, GT, and Weinfurter, PRL 2009]

[cold atoms: Lücke *et al.*, Science 2011; Hamley *et al.*, Science 2011; C. Gross *et al.*, Nature 2011]

Dicke states are useful because they ...

- ... possess strong multipartite entanglement, like GHZ states.

[GT, JOSAB 2007.]

- ... are optimal for quantum metrology, similarly to GHZ states.

[Hyllus *et al.*, PRA 2012; Lücke *et al.*, Science 2011; GT, PRA 2012; GT and Apellaniz, JPHYSA, 2014.]

- ... are macroscopically entangled, like GHZ states.

[Fröwis, Dür, PRL 2011]

Spin Squeezing Inequality for Dicke states

- Let us rewrite the third inequality

$$\langle J_k^2\rangle + \langle J_l^2\rangle - \frac{N}{2} \le (N-1)(\Delta J_m)^2.$$

- It detects states close to Dicke states since

$$\langle J_x^2 + J_y^2\rangle = \frac{N}{2}\left(\frac{N}{2}+1\right) = \mathrm{max.},$$
$$\langle J_z^2\rangle = 0.$$

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Multipartite entanglement in spin squeezing

- We consider pure k-producible states of the form

$$|\Psi\rangle = \otimes_{l=1}^{M} |\psi_l\rangle,$$

where $|\psi_l\rangle$ is the state of at most k qubits.

Extreme spin squeezing

The spin-squeezing criterion for k-producible states is

$$(\Delta J_z)^2 \geqslant J_{\max} F_{\frac{k}{2}} \left(\frac{\sqrt{\langle J_x \rangle^2 + \langle J_y \rangle^2}}{J_{\max}} \right),$$

where $J_{\max} = \frac{N}{2}$ and we use the definition

$$F_j(X) := \frac{1}{j} \min_{\frac{\langle j_x \rangle}{j} = X} (\Delta j_z)^2.$$

[Sørensen and Mølmer, Phys. Rev. Lett. 86, 4431 (2001);
experimental test: C. Gross, T. Zibold, E. Nicklas, J. Esteve, and M. K. Oberthaler,

Multipartite entanglement around Dicke states

- Measure the same quantities as before

$$(\Delta J_z)^2$$

and

$$\langle J_x^2 + J_y^2 \rangle.$$

- In contrast, for the original spin-squeezing criterion we measured $(\Delta J_z)^2$ and $\langle J_x \rangle^2 + \langle J_y \rangle^2$.

- Pioneering work: linear condition of Luming Duan, Phys. Rev. Lett. (2011). See also Zhang, Duan, New. J. Phys. (2014).

Multipartite entanglement

- Sørensen-Mølmer condition for k-producible states:

$$(\Delta J_z)^2 \geqslant J_{\max} F_{\frac{k}{2}} \left(\frac{\sqrt{\langle J_x \rangle^2 + \langle J_y \rangle^2}}{J_{\max}} \right).$$

- Combine it with

$$\langle J_x^2 + J_y^2 \rangle \leqslant J_{\max}(\tfrac{k}{2} + 1) + \langle J_x \rangle^2 + \langle J_y \rangle^2,$$

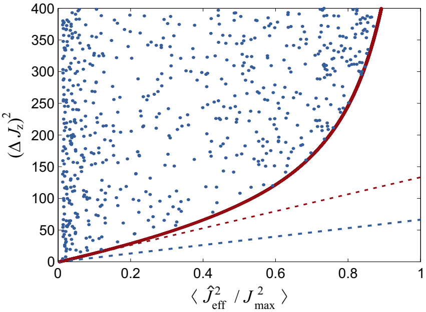
which is true for pure k-producible states. (Remember, $J_{\max} = \frac{N}{2}$.)

Condition for entanglement detection around Dicke states

$$(\Delta J_z)^2 \geqslant J_{\max} F_{\frac{k}{2}} \left(\frac{\sqrt{\langle J_x^2 + J_y^2 \rangle - J_{\max}(\frac{k}{2} + 1)}}{J_{\max}} \right).$$

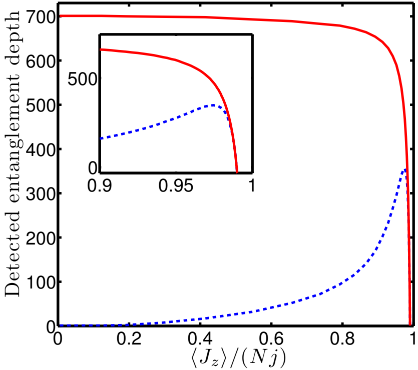
Due to convexity properties of the expression, this is also valid to mixed separable states. [Lücke *et. al*, PRL 2014.]

Concrete example



- $N = 8000$ particles, and $J_{\mathrm{eff}} = J_x^2 + J_y^2$.
- Red curve: boundary for 28-particle entanglement.
- Blue dashed line: linear condition given in [L.-M. Duan, Phys. Rev. Lett. 107, 180502 (2011).]
- Red dashed line: tangent of our curve.

Comparison of criteria



spin squeezing, $N = 1000$ spin-$\frac{1}{2}$ particles, 10 particles decohered

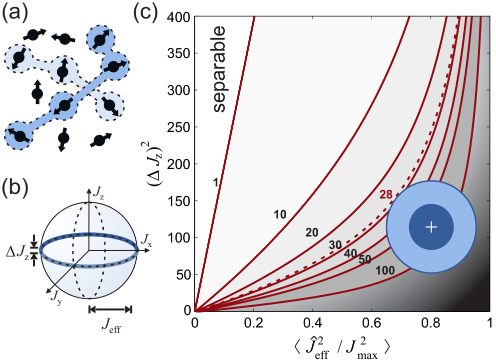
(solid) Our criterion.

(dashed) the Sørensen-Mølmer criterion.

[Vitagliano, Apellaniz, Kleinmann, Lücke, Klempt, NJP 19, 013027 (2017).]

Experimental results

- Bose-Einstein condensate, 8000 particles. 28-particle entanglement is detected.



Giuseppe Vitagliano

[Lücke *et al.*, Phys. Rev. Lett. 112, 155304 (2014)]

Criteria for $j > 1/2$

- Dicke states of particles with a spin larger than $1/2$ have been created.
- See, for example, T. M. Hoang, M. Anquez, M. J. Boguslawski, H. M. Bharath, B. A. Robbins, and M. S. Chapman, arXiv:1512.06766 (2015).

[Vitagliano *et al.*, NJP 2017.]

- Note also alternative methods for entanglement detection for large spins.

[G. Vitagliano et al., Phys. Rev. A 89, 032307 (2014); G. Vitagliano et al., Phys. Rev. Lett. 107, 240502 (2011).]

Criterion for any j

- Sørensen-Mølmer condition for k-producible states with $J = kj$

$$(\Delta J_z)^2 \geqslant J_{\max} F_J\left(\frac{\sqrt{\langle J_x\rangle^2 + \langle J_y\rangle^2}}{J_{\max}}\right).$$

- Combine it with

$$\frac{\sqrt{\left\langle J_y\right\rangle^2 + \langle J_z\rangle^2}}{Nj} \geq \sqrt{\frac{\left\langle J_y^2 + J_z^2\right\rangle - Nj\,(kj+1)}{N(N-k)j^2}}$$

which is true for pure k-producible states. (Remember, $J_{\max} = Nj$.)

Condition for entanglement detection around Dicke states

$$(\Delta J_x)^2 \geq Nj F_J\left(\sqrt{\frac{\left\langle J_y^2 + J_z^2\right\rangle - Nj(kj+1)}{N(N-k)j^2}}\right).$$

[Vitagliano *et. al*, NJP 2017.]

Spin squeezing parameters

- The condition based on the tangent could be obtained with perturbation theory as

$$\xi^2 := (kj+1)\frac{2(N-k)j(\Delta J_x)^2}{\left\langle J_y^2 + J_z^2 \right\rangle - Nj(kj+1)} \geq 1.$$

- Similar condition for spin squeezing is

$$\xi_{\rm SM}^2 := (kj+1)\frac{2Nj(\Delta J_x)^2}{\left\langle J_y \right\rangle^2 + \langle J_z \rangle^2} \geq 1.$$

[Vitagliano, Apellaniz, Kleinmann, Lücke, Klempt, Tóth, NJP 19, 013027 (2017).]

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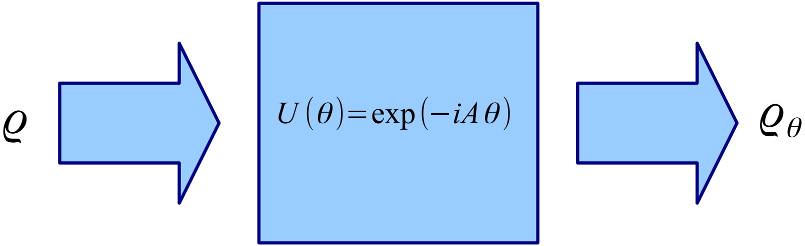
Quantum metrology

- Fundamental task in metrology



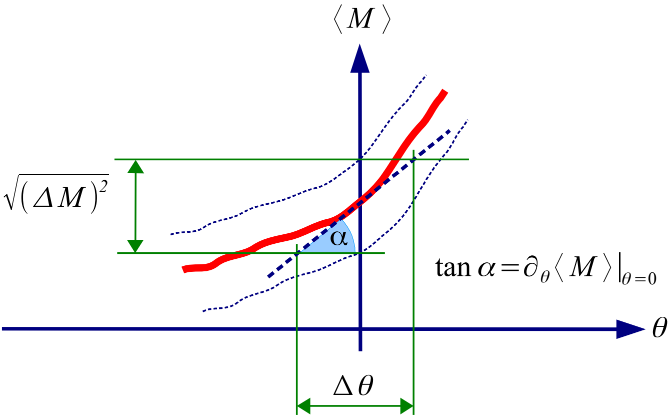
- We have to estimate θ in the dynamics

$$U = \exp(-iA\theta).$$

Precision of parameter estimation

- Measure an operator M to get the estimate θ. The precision is

$$(\Delta\theta)^2 = \frac{(\Delta M)^2}{|\partial_\theta \langle M \rangle|^2}.$$



The quantum Fisher information

- Cramér-Rao bound on the precision of parameter estimation

$$(\Delta\theta)^2 \geq \frac{1}{F_Q[\varrho, A]}, \qquad (\Delta\theta)^{-2} \leq F_Q[\varrho, A].$$

  where $F_Q[\varrho, A]$ is the quantum Fisher information.

- The quantum Fisher information is

$$F_Q[\varrho, A] = 2 \sum_{k,l} \frac{(\lambda_k - \lambda_l)^2}{\lambda_k + \lambda_l} |\langle k|A|l\rangle|^2,$$

  where $\varrho = \sum_k \lambda_k |k\rangle\langle k|$.

The quantum Fisher information vs. entanglement

- For separable states

$$F_Q[\varrho, J_l] \leq N.$$

[Pezze, Smerzi, Phys. Rev. Lett. 102, 100401 (2009); Hyllus, Gühne, Smerzi, Phys. Rev. A 82, 012337 (2010)]

- For states with at most k-particle entanglement (k is divisor of N)

$$F_Q[\varrho, J_l] \leq kN.$$

[P. Hyllus *et al.*, Phys. Rev. A 85, 022321 (2012); GT, Phys. Rev. A 85, 022322 (2012)].

⇒ Talk of Manuel Gessner about how to improve these conditions.

- Macroscopic superpositions (e.g, GHZ states, Dicke states)

$$F_Q[\varrho, J_l] \propto N^2$$

[F. Fröwis, W. Dür, New J. Phys. 14 093039 (2012).]

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Metrology with Dicke states

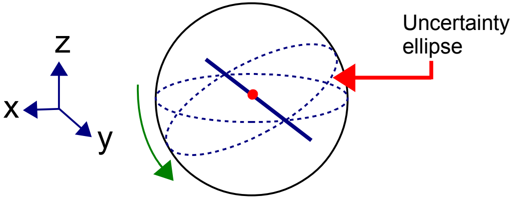
- For Dicke state

$$\langle J_l \rangle = 0, l = x, y, z, \quad \langle J_z^2 \rangle = 0, \quad \langle J_x^2 \rangle = \langle J_y^2 \rangle = \text{large}.$$

- Linear metrology

$$U = \exp(-iJ_y\theta).$$

- Measure $\langle J_z^2 \rangle$ to estimate θ. (We cannot measure first moments, since they are zero.)

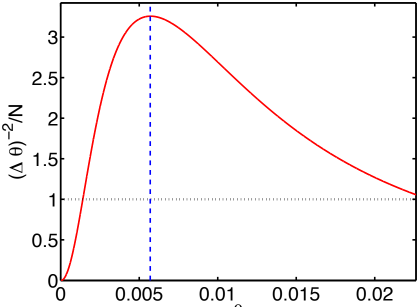


Metrology with Dicke states II

We measure $\langle J_z^2 \rangle$ to estimate θ. The precision is given by the error-propagation formula

$$(\Delta\theta)^2 = \frac{(\Delta J_z^2)^2}{|\partial_\theta \langle J_z^2 \rangle|^2}.$$

- Precision as a function of θ for some noisy Dicke state



Formula for maximal precision

Parameter value for the maximum

$$\tan^2 \theta_{\rm opt} = \sqrt{\frac{(\Delta J_z^2)^2}{(\Delta J_x^2)^2}}.$$

Consistency check: for the noiseless Dicke state we have $(\Delta J_z^2)^2 = 0$, hence $\theta_{\rm opt} = 0$.

Iagoba Apellaniz

[I. Apellaniz, B. Lücke, J. Peise, C. Klempt, GT, New J. Phys. 17, 083027 (2015).]

Formula for maximal precision II

Maximal precision with a closed formula

$$(\Delta\theta)^2_{\rm opt} = \frac{2\sqrt{(\Delta J_z^2)^2(\Delta J_x^2)^2}+4\langle J_x^2\rangle-3\langle J_y^2\rangle-2\langle J_z^2\rangle(1+\langle J_x^2\rangle)+6\langle J_zJ_x^2J_z\rangle}{4(\langle J_x^2\rangle-\langle J_z^2\rangle)^2}.$$

- Given in terms of collective observables, like spin-squeezing criteria.
- Metrological usefulness can be verified without carrying out the metrological task.

[I. Apellaniz, B. Lücke, J. Peise, C. Klempt, GT, New J. Phys. 17, 083027 (2015).]

Experimental test of our formula

- Trying the bound for the experimental data for $N = 7900$ particles

$$\langle J_z^2 \rangle = 112 \pm 31, \qquad \langle J_z^4 \rangle = 40 \times 10^3 \pm 22 \times 10^3,$$
$$\langle J_x^2 \rangle = 6 \times 10^6 \pm 0.6 \times 10^6, \quad \langle J_x^4 \rangle = 6.2 \times 10^{13} \pm 0.8 \times 10^{13}.$$

- Hence, we obtain

$$\frac{(\Delta\theta)^{-2}_{\mathrm{opt}}}{N} \geq 3.7 \pm 1.5.$$

- Remember, for states for at most k-particle entanglement we have

$$(\Delta\theta)^{-2} \leq F_Q[\varrho, J_l] \leq kN.$$

- Thus, four-particle entanglement is detected for this particular measurement.

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Most important characteristics used for estimation

The quantum Fisher information is the convex roof of the variance

$$F_Q[\varrho, A] = 4 \min_{p_k, \Psi_k} \sum_k p_k (\Delta A)^2{}_k,$$

where

$$\varrho = \sum_k p_k |\Psi_k\rangle\langle\Psi_k|.$$

[GT, D. Petz, Phys. Rev. A 87, 032324 (2013); S. Yu, arXiv1302.5311 (2013);
GT, I. Apellaniz, J. Phys. A: Math. Theor. 47, 424006 (2014)]

- Thus, it is similar to entanglement measures that are also defined by convex roofs.

Measure the quantum Fisher information

- We would like to measure the quantum Fisher information.
- For systems in thermal equilibrium, there are methods, e.g., Hauke et al., Nat. Phys. 12, 778 (2016).

$$F_Q(T) = \frac{4}{\pi} \int_0^\infty \mathrm{d}\omega \tanh\left(\frac{\omega}{2T}\right) \chi''(\omega, T)$$

- This method needs a lot of measurements.
- We have systems not in thermal equilibrium, and want to measure few operators.

Legendre transform

- Optimal linear lower bound on a convex function $g(\varrho)$ based on an operator expectation value $w = \langle W \rangle_\varrho = \mathrm{Tr}(W\varrho)$

$$g(\varrho) \geq rw - const.,$$

where $w = \mathrm{Tr}(\varrho W)$.

- For every r there is a "*const.*" that makes the relation an optimal linear lower bound.

- How large is "*const.*"? It can be obtained as

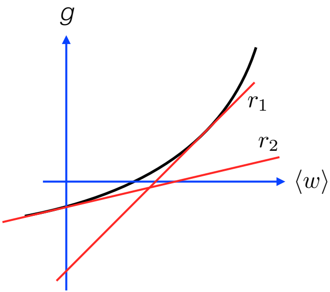
$$g(\varrho) \geq \mathcal{B}(w) := rw - \hat{g}(rW),$$

where $\hat{g}$ is the Legendre transform

$$\hat{g}(W) = \sup_\varrho [\langle W \rangle_\varrho - g(\varrho)].$$

[O. Gühne, M. Reimpell, and R. F. Werner, PRL 98, 110502 (2007);
J. Eisert, F. G. S. L. Brandao, and K. M. R. Audenaert, NJP 9, 46 (2007).]

Legendre transform II



- Tight lower bound can be obtained if we optimize over r as

$$g(\varrho) \geq \mathcal{B}(w) := \sup_r \left[rw - \hat{g}\left(rW \right) \right],$$

where again $w = \mathrm{Tr}(\varrho W)$.

- The quantum Fisher information is given as a convex roof. Enough to carry out an optimization over pure states

$$\hat{g}(W) = \sup_\Psi [\langle W \rangle_\Psi - g(\Psi)].$$

Legendre transform VI

- For our case, the Legendre transform is

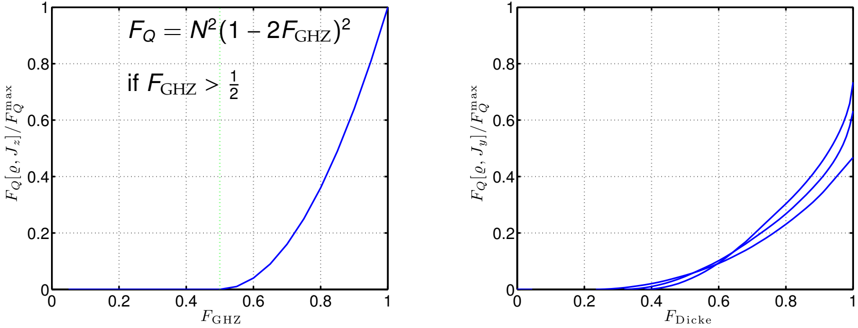
$$\hat{\mathcal{F}}_{\mathrm{Q}}(W) = \sup_{\Psi}[\langle W - 4J_l^2 \rangle_\Psi + 4 \langle J_l \rangle_\Psi^2].$$

- With further simplifications, an optimization over a single real variable is needed

$$\hat{\mathcal{F}}_{\mathrm{Q}}(W) = \sup_{\mu} \left\{ \lambda_{\max} \left[W - 4(J_l - \mu)^2 \right] \right\}.$$

Witnessing the quantum Fisher information based on the fidelity

- Let us bound the quantum Fisher information based on some measurements. First, consider small systems.
  [See also Augusiak *et al.*, 1506.08837.]



Quantum Fisher information vs. Fidelity with respect to (a) GHZ states and (b) Dicke states for $N = 4, 6, 12$.

[Apellaniz *et al.*, arXiv:1511.05203.]

The GHZ state case

- A similar relation for the Fisher information is for metrology with GHZ states

$$F(\theta) = \frac{V^2 N^2 \sin^2 N\theta}{1 - V^2 \cos^2 N\theta},$$

where the visibility is defined

$$V = (1 - 2\mathrm{Fidelity})^2 \text{ for Fidelity} \geq 1/2.$$

The maximum is

$$F_{\max} = V^2 N^2.$$

- Note that this is an equality, rather than an inequality since the noise considered is

$$\varrho_{\mathrm{noise}} = \frac{1}{2}(|000..00\rangle\langle 000..00| + |111..11\rangle\langle 111..11|).$$

[L. Pezze, Y. Li, W. Li, and A. Smerzi, PNAS 113, 11459 (2016).]

Bounding the qFi based on collective measurements

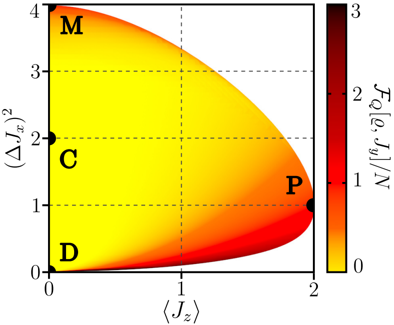
Bound for the quantum Fisher information for spin squeezed states (Pezze-Smerzi bound)

$$F[\varrho, J_y] \geq \frac{\langle J_z \rangle^2}{(\Delta J_x)^2}.$$

[Pezze, Smerzi, Phys. Rev. Lett. 102, 100401 (2009).]

Bounding the qFi based on collective measurements II

- Optimal bound for the quantum Fisher information $F_Q[\varrho, J_y]$ for spin squeezing for $N = 4$ particles

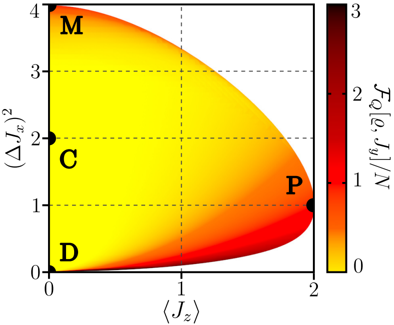


P=fully polarized state, D=Dicke state, C=completely mixed state, M=mixture of $|00..000\rangle_x$ and $|11..111\rangle_x$

[Apellaniz, Kleinman, Gühne, GT, arXiv:1511.05203.]

Bounding the qFi based on collective measurements III

- Optimal bound for the quantum Fisher information $F_Q[\varrho, J_y]$ for spin squeezing for $N = 4$ particles

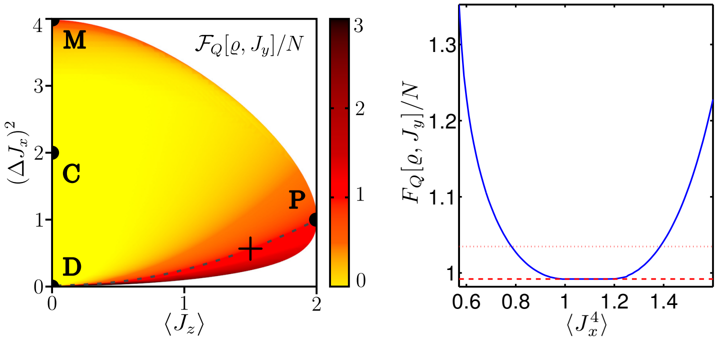


On the bottom part of the figure ($(\Delta J_x)^2 < 1$) the bound is very close to the Pezze-Smerzi bound!

[Apellaniz, Kleinman, Gühne, GT, arXiv:1511.05203.]

Bounding the qFi based on collective measurements IV

- The bound can be obtained if additional expectation value, i.e., $\langle J_x^2 \rangle$ is measured, or we assume symmetry:



[Apellaniz, Kleinman, Gühne, GT, arXiv:1511.05203.]

Spin squeezing experiment

- Experiment with $N = 2300$ atoms,

$$\xi_s^2 = -8.2\mathrm{dB} = 10^{-8.2/10} = 0.1514.$$

[C. Gross, T. Zibold, E. Nicklas, J. Esteve, and M. K. Oberthaler, Nature 464, 1165 (2010).]

- We choose

$$\langle J_z \rangle = \alpha \frac{N}{2},$$

with $\alpha = 0.85$. (Almost fully polarized.)

- The Pezze-Smerzi bound is:

$$\frac{\mathcal{F}_Q[\varrho_N, J_y]}{N} \geq \frac{1}{\xi_s^2} = 6.605,$$

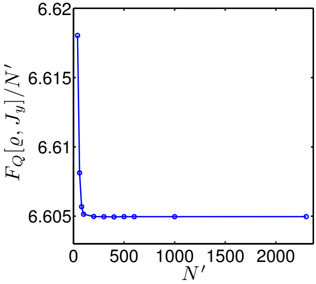
[Pezze, Smerzi, Phys. Rev. Lett. 102, 100401 (2009).]

Spin squeezing experiment

- Solution starting with small systems, using

$$\langle J_z \rangle = \frac{N'}{2}\alpha,$$
$$(\Delta J_x)^2 = \xi_s^2 \frac{N'}{4}\alpha^2.$$

- We get 6.605!!



- Proof that the formula is optimal, and that our method works.

New bounds on the quantum Fisher information

- Lower bound on the quantum Fisher information

$$(\Delta J_l)^2 - \tfrac{1}{4} F_Q[\varrho, J_l] \le \frac{N^2}{2}[1 - \mathrm{Tr}(\varrho^2)].$$

- Lower bound on the average quantum Fisher information

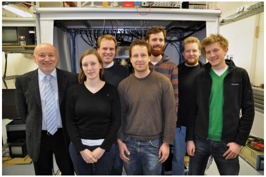
$$\sum_k \left\{(\Delta A_k)^2 - \tfrac{1}{4} F_Q[\varrho, A_k]\right\} = 2\Big[S_{\mathrm{lin}}(\varrho) + H(\varrho) - 1\Big].$$

where A_k are SU(d) generators and

$$H(\varrho) = 2\sum_{k,l} \frac{\lambda_k \lambda_l}{\lambda_k + \lambda_l} = 1 + 2\sum_{k \neq l} \frac{\lambda_k \lambda_l}{\lambda_k + \lambda_l}.$$

[G. Tóth, arXiv:1701.07461.]

Project participants



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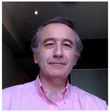

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Siegen

Summary

- Detection of multipartite entanglement and metrological usefulness close to Dicke states, by measuring collective quantities only.

Lücke, Peise, Vitagliano, Arlt, Santos, GT, Klempt, PRL 112, 155304 (2014);
Vitagliano, Apellaniz, Kleinmann, Lücke, Klempt, GT, NJP 19, 013027 (2017);
Apellaniz, Lücke, Peise, Klempt, GT, NJP 17, 083027 (2015);
Apellaniz, Kleinmann, Gühne, Tóth, arxiv: arXiv:1511.05203.

THANK YOU FOR YOUR ATTENTION!

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