Activating hidden metrological usefulness

Géza Tóth[1,2,3,4], Tamás Vértesi[5], Paweł Horodecki[6,7], Ryszard Horodecki[8]

[1]Theoretical Physics, University of the Basque Country (UPV/EHU), Bilbao, Spain
[2]Donostia International Physics Center (DIPC), San Sebastián, Spain
[3]IKERBASQUE, Basque Foundation for Science, Bilbao, Spain
[4]Wigner Research Centre for Physics, Budapest, Hungary
[5]Institute for Nuclear Research, Hungarian Academy of Sciences, Debrecen, Hungary
[6]International Centre for Theory of Quantum Technologies, University of Gdansk, Poland
[7]Faculty of Applied Physics and Mathematics, National Quantum Information Centre, Gdansk University of Technology, Gdansk, Poland
[8]Institute of Theoretical Physics and Astrophysics, National Quantum Information Centre, Faculty of Mathematics, Physics and Informatics, University of Gdansk, Poland

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Photos

Tamás Vértesi

Paweł Horodecki

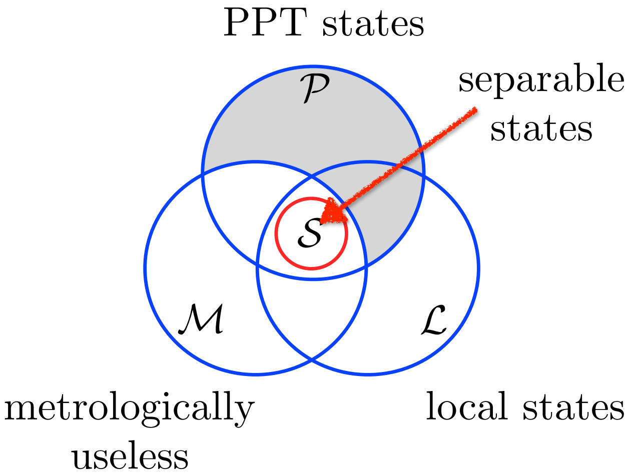
Ryszard Horodecki

Outline

What are entangled states useful for?

- Entangled states are useful, but not all of them are useful for some task.
- Entanglement is needed for beating the shot-noise limit in quantum metrology.
- Intriguing questions:
  - Can a quantum state become useful metrologically, if an ancilla or a second copy is added?
  - How to find the local Hamiltonian, for which a quantum state is the most useful compared to separable states?

What are entangled states useful for?



Tóth and Vértesi, Phys. Rev. Lett. 2018.

Outline

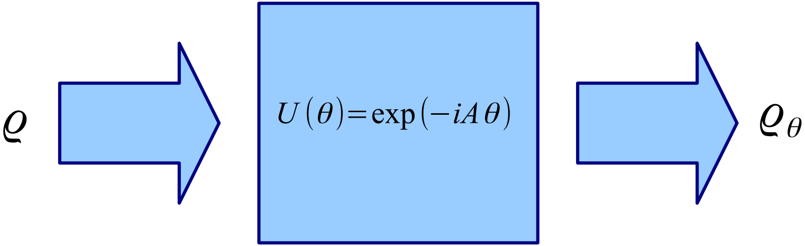
1. **Motivation**
   - What are entangled states useful for?

2. **Bacground**
   - Quantum Fisher information
   - Recent findings on the quantum Fisher information

3. **Metrological gain and the optimal local Hamiltonian**
   - Metrological usefulness of a quantum state.
   - Activation of metrological usefulness
   - Optimal local Hamiltonian
   - All bipartite pure entangled states are useful

Quantum metrology

- Fundamental task in metrology



- We have to estimate θ in the dynamics

$$U = \exp(-iA\theta).$$

The quantum Fisher information

- Cramér-Rao bound on the precision of parameter estimation

$$(\Delta\theta)^2 \geq \frac{1}{mF_Q[\varrho, A]},$$

where where m is the number of independent repetitions and $F_Q[\varrho, A]$ is the quantum Fisher information.

- The quantum Fisher information is

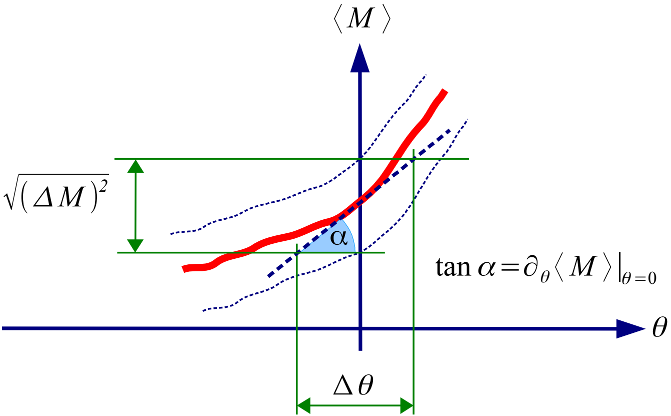
$$F_Q[\varrho, A] = 2\sum_{k,l} \frac{(\lambda_k - \lambda_l)^2}{\lambda_k + \lambda_l} |\langle k|A|l\rangle|^2,$$

where $\varrho = \sum_k \lambda_k |k\rangle\langle k|$.

Error propagation formula

- Measure an operator M to get the estimate θ. The error propagation formula is

$$(\Delta\theta)^2_M = \frac{(\Delta M)^2}{|\partial_\theta \langle M \rangle|^2}.$$



Relation between $(\Delta\theta)^2$ and the error propagation formula $(\Delta\theta)^2_M$

- The relation

$$(\Delta\theta)^2 \geq \frac{1}{m}(\Delta\theta)^2{}_{M_{\rm opt}}$$

holds, where m is the number of independent repetitions and $M_{\rm opt}$ is the optimal observable.

- The relation can be saturated if m is large and the distribution fulfills certain requirements.

L. Pezze, A. Smerzi, M. K. Oberthaler, R. Schmied, and P. Treutlein, Rev. Mod. Phys. 2018.

- Morover,

$$(\Delta\theta)^2{}_{M_{\rm opt}} = \frac{1}{F_Q[\varrho, A]}.$$

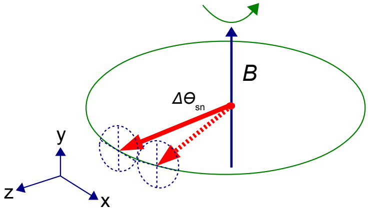
M. Hotta and M. Ozawa, Phys. Rev. A 2004; B. M. Escher, arXiv:1212.2533; F. Fröwis, R. Schmied, and N. Gisin, Phys. Rev. A 2015. For a summary, see, e.g., the Supplemental Material of Tóth, Vértesi, Horodecki, Horodecki, PRL 2020.

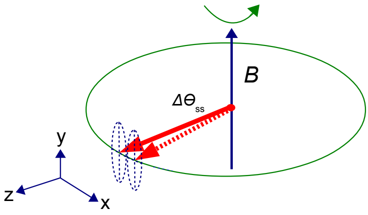
Special case $A = J_l$

- The operator A is defined as

$$A = J_l = \sum_{n=1}^{N} j_l^{(n)}, \quad l \in \{x, y, z\}.$$

- Magnetometry with a linear interferometer





Outline

1. **Motivation**
   - What are entangled states useful for?
2. **Bacground**
   - Quantum Fisher information
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   - Activation of metrological usefulness
   - Optimal local Hamiltonian
   - All bipartite pure entangled states are useful

Properties of the Fisher information

Many bounds on the quantum Fisher information can be derived from these simple properties:

- For pure states, it equals four times the variance,

$$F[|\Psi\rangle\langle\Psi|, A] = 4(\Delta A)^2{}_\Psi.$$

- For mixed states, it is convex.

The quantum Fisher information vs. entanglement

- For separable states

$$F_Q[\varrho, J_l] \le N, \qquad l = x, y, z.$$

[Pezze, Smerzi, Phys. Rev. Lett. 102, 100401 (2009); Hyllus, Gühne, Smerzi, Phys. Rev. A 82, 012337 (2010)]

- For states with at most k-particle entanglement (k is divisor of N)

$$F_Q[\varrho, J_l] \le kN.$$

[P. Hyllus *et al.*, Phys. Rev. A 85, 022321 (2012); GT, Phys. Rev. A 85, 022322 (2012)].

- Macroscopic superpositions (e.g, GHZ states, Dicke states)

$$F_Q[\varrho, J_l] \propto N^2,$$

[F. Fröwis, W. Dür, New J. Phys. 14 093039 (2012).]

Behavior under noise

Even a little uncorrelated local noise leads to shot-noise scaling above a certain particle number.

The quantum Fisher information is the convex roof over the purifications of the dynamics

$$F_Q[\varrho_\theta] = \min_{|\Psi_\theta\rangle} F_Q[|\Psi_\theta\rangle],$$

where the purification is $|\Psi_\theta\rangle$ is related to the state as

$$\varrho_\theta = \mathrm{Tr}_E(|\Psi_\theta\rangle\langle\Psi_\theta|).$$

[R. Demkowicz-Dobrzański, J. Kołodyński, M. Guţă, Nature Comm. 2012.]

Most important characteristics used for estimation

The quantum Fisher information is the convex roof of the variance

$$F_Q[\varrho, A] = 4 \min_{p_k, \Psi_k} \sum_k p_k (\Delta A)^2{}_k,$$

where

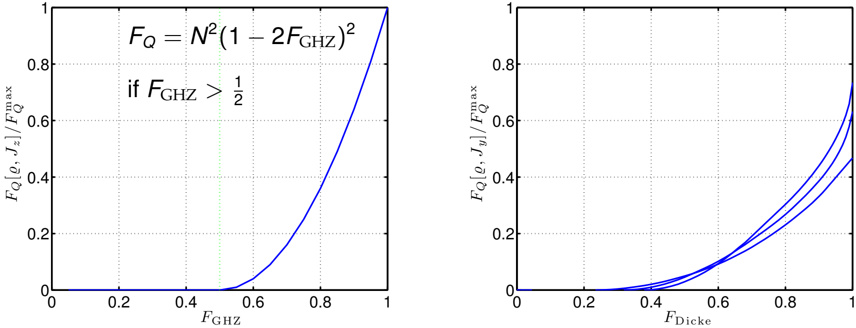
$$\varrho = \sum_k p_k |\Psi_k\rangle\langle\Psi_k|.$$

[GT, D. Petz, Phys. Rev. A 87, 032324 (2013); S. Yu, arXiv1302.5311 (2013); GT, I. Apellaniz, J. Phys. A: Math. Theor. 47, 424006 (2014)]

- Extended convexity for non-unitary dynamics.
  [S. Alipour, A. T. Rezakhani, Phys. Rev. A 91, 042104 (2015).]

Witnessing the quantum Fisher information based on few measurements

- Let us bound the quantum Fisher information based on some measurements.



Quantum Fisher information vs. Fidelity with respect to (a) GHZ states and (b) Dicke states for $N = 4, 6, 12$.

[Apellaniz *et al.*, Phys. Rev. A 2017]

Continuity of QFI and QFI for symmetric states

- Arbitrarily small entanglement can be used to get close to Heisenberg scaling.
- The difference between the QFI of two states can be bounded by the distance of the two states.
- Bound on the QFI with the geometric measure of entanglement.

[R. Augusiak, J. Kołodyński, A. Streltsov, M. N. Bera, A. Acín, M. Lewenstein, PRA 2016]

- Continuity in the non-unitary case:

[A. T. Rezakhani, S. Alipour, M. Hassani, PRA 2019]

- Random pure states of distinguishable particles typically do not lead to super-classical scaling of precision.
- Random states from the symmetric subspace typically achieve the optimal Heisenberg scaling.

[M. Oszmaniec, R. Augusiak, C. Gogolin, J. Kołodyński, A. Acín, M. Lewenstein, PRX 2016]

Metrologically useful PPT states

- Bound entangled states with PPT and some non-PPT partitions.
- Violates an entanglement criterion with three QFI terms.

  [P. Hyllus, W. Laskowski, R. Krischek, C. Schwemmer, W. Wieczorek, H. Weinfurter, L. Pezze, and A. Smerzi, PRA 85, 022321 (2012).]

- Non-unlockable bound entangled states with PPT and some non-PPT partitions.
- Violates the criterion with a single QFI term, better than shot-noise limit.

  [Ł. Czekaj, A. Przysiężna, M. Horodecki, P. Horodecki, Phys. Rev. A 92, 062303 (2015).]

- Bipartite PPT entangled states can be useful for metrology, and they are close to be maximally useful for large dimensions.

  [Tóth and Vértesi, Phys. Rev. Lett. 2018;
  K. F. Pál, G. Tóth, E. Bene. T. Vértesi, arxiv 2020.]

Outline

Metrological usefulness

- Metrological gain for a given Hamiltonian

$$g_{\mathcal{H}}(\varrho) = \frac{\mathcal{F}_Q[\varrho, \mathcal{H}]}{\mathcal{F}_Q^{(\mathrm{sep})}(\mathcal{H})},$$

  where $\mathcal{F}_Q^{(\mathrm{sep})}(\mathcal{H})$ is the maximum of the QFI for separable states.

- Metrological gain optimized over all local Hamiltonians

$$g(\varrho) = \max_{\mathrm{local}\mathcal{H}} g_{\mathcal{H}}(\varrho) = \max_{\mathrm{local}\mathcal{H}} \frac{\mathcal{F}_Q[\varrho, \mathcal{H}]}{\mathcal{F}_Q^{(\mathrm{sep})}(\mathcal{H})}.$$

- The metrological gain is convex in the state.
  [G. Toth, T. Vertesi, P. Hordecki, R. Horodecki, PRL 2020.]

- We would like to detmine g.

Metrological usefulness II

- So we would like optimize over local $\mathcal{H}$ the expression

$$g(\varrho) = \max_{\text{local}\mathcal{H}} \frac{\mathcal{F}_Q[\varrho, \mathcal{H}]}{\mathcal{F}_Q^{(\text{sep})}(\mathcal{H})}.$$

- First observation: we really optimize the QFI over $\mathcal{H}$, but we normalize it with something meaningful.
- This is needed, since otherwise $\mathcal{H}' = 100\mathcal{H}$ would be better than $\mathcal{H}$.
- Second observation: difficult task, since both the numerator and the denominator depend on $\mathcal{H}$.

Maximally entangled state

- It is a difficult task to obtain $g(\varrho)$ and the optimal local Hamiltonian for any ϱ.

- As a first step, we consider the $d \times d$ maximally entangled state, which is defined as

$$|\Psi^{(\mathrm{me})}\rangle = \frac{1}{\sqrt{d}} \sum_{k=1}^{d} |k\rangle|k\rangle.$$

- Due to the symmetry of the state, the optimal Hamiltonian is

$$\mathcal{H}^{(\mathrm{me})} = D \otimes \mathbb{1} + \mathbb{1} \otimes D,$$

where the diagonal matrix D is given as

$$D = \mathrm{diag}(+1, -1, +1, -1, ...).$$

Maximally entangled state II

- For the 3×3-case, we consider the noisy quantum state

$$\varrho_{AB}^{(p)} = (1-p)|\Psi^{(\mathrm{me})}\rangle\langle\Psi^{(\mathrm{me})}| + p\mathbb{1}/d^2,$$

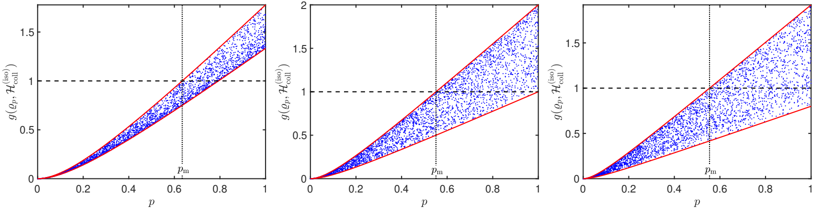
which is useful if

$$p < \frac{25-\sqrt{177}}{32} \approx 0.3655.$$

- Note that it is entangled if

$$p < \frac{2}{3}.$$

Maximally entangled state III



3×3 4×4 5×5

Metrology with isotropic states, p is the weight of the maximally entangled state,

$$\mathcal{H}_{\rm coll}^{\rm (iso)}(\mathcal{H}^{\rm (iso)}) = \mathcal{H}^{\rm (iso)} \otimes \mathbb{1} + \mathbb{1} \otimes (\mathcal{H}^{\rm (iso)})^*,$$

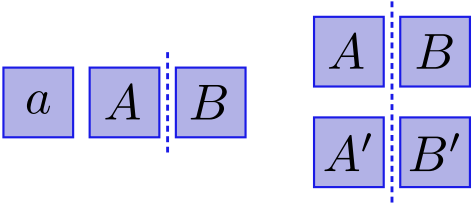
(- - - -) Separable states.

(• • •) Isotropic states for two-body Hamiltonians $\mathcal{H}_{\rm coll}^{\rm (iso)}(\mathcal{H}^{\rm (iso)})$, where $\mathcal{H}^{\rm (iso)}$ are chosen randomly.

$(\cdot \cdot \cdot \cdot \cdot)$ Isotropic states with a larger p are useful for metrology.

Outline

Activation by an ancilla qubit or a second copy



(a) (b)

(a) An ancilla ("*a*") is added to bipartite state ϱ_{AB}.

(b) An additional copy or a different state is added to the state.

Activation by an ancilla qubit

- Now we consider the previous state, after a pure ancilla qubit is added

$$\varrho^{(\mathrm{anc})} = |0\rangle\langle 0|_a \otimes \varrho_{AB}^{(p)}.$$

- Then, with the operator

$$\mathcal{H}^{(\mathrm{anc})} = 1.2 C_{aA} \otimes \mathbb{1}_B + \mathbb{1}_{aA} \otimes D_B,$$

  where an operator acting on the ancilla and A is

$$C_{aA} = \frac{9}{20}(2\sigma_x + \sigma_z)_a \otimes |0\rangle\langle 0|_a + \mathbb{1}_a \otimes (|2\rangle\langle 2|_a - |1\rangle\langle 1|_a),$$

  we have $g_{\mathcal{H}^{(\mathrm{anc})}}(\varrho^{(\mathrm{anc})}) > 1$ if

$$p < 0.3752.$$

- Hence larger part of the noisy maximally entangled states are useful in the case with the ancilla (For a single copy, the limit was $p<0.3655$.)

Activation by a second copy

- We consider now two copies of the noisy 3×3 maximally entangled state

$$\varrho^{(\mathrm{tc})} = \varrho_{AB}^{(p)} \otimes \varrho_{A'B'}^{(p)}.$$

- Then, with the two-copy operator

$$\mathcal{H}^{(\mathrm{tc})} = D_A \otimes D_{A'} \otimes \mathbb{1}_{BB'} + \mathbb{1}_{AA'} \otimes D_B \otimes D_{B'},$$

  we have $g_{\mathcal{H}^{(\mathrm{tc})}}(\varrho^{(\mathrm{tc})}) > 1$ if

$$p < 0.4164.$$

- Hence larger part of the noisy maximally entangled states are useful in the two-copy case, than with a single copy. (For a single copy, the limit was $p < 0.3655$.)

Outline

Method for finding the optimal local Hamiltonian - Exisiting work for qubits

- For qubits, the local Hamiltonians with eigenvalues $+1$ and -1 differ from each other by local unitaries

$$\mathcal{H} = U_1\sigma_z U_1^\dagger \otimes \mathbb{1} + \mathbb{1} \otimes U_2\sigma_z U_2^\dagger.$$

- It is possible to obtain bounds on the quantum Fisher information.

- All pure two-qubit entangled states are useful, while not all pure multi-qubit entangled states are useful.

[P. Hyllus, O. Gühne, and A. Smerzi, Phys. Rev. A 82, 012337 (2010).]

- When looking at $\mathcal{F}_Q/\mathcal{F}_Q^{(\mathrm{sep})}$, the value of $\mathcal{F}_Q^{(\mathrm{sep})}$ does not depend on the particular Hamiltonian. For instance for spin operators $\mathcal{F}_Q^{(\mathrm{sep})} = N$.

[L. Pezze and A. Smerzi, Phys. Rev. Lett. 2009.]

Method for finding the optimal local Hamiltonian for qudits with $d > 2$

- The case of qudits is more complicated than the case of qubits, since the local Hamiltonians cannot be converted to each other by unitaries.

- We need to maximize $\mathcal{F}_Q[\varrho, \mathcal{H}]$ over $\mathcal{H}$ for a given ϱ.
  However, $\mathcal{F}_Q$ convex in $\mathcal{H}$, maximizing it over $\mathcal{H}$ is a difficult task.

- Instead of the quantum Fisher information, let us consider the error propagation formula

$$(\Delta\theta)^2{}_M = \frac{(\Delta M)^2}{\langle i[M, \mathcal{H}]\rangle^2},$$

  which provides a bound on the quantum Fisher information

$$\mathcal{F}_Q[\varrho, \mathcal{H}] \geq 1/(\Delta\theta)^2{}_M.$$

  We will now minimize $(\Delta\theta)^2{}_M$.

Method for finding the optimal Hamiltonian II

- To be more specific, we use

$$\mathcal{F}_Q[\varrho, \mathcal{H}] = \max_M \frac{\langle i[M, \mathcal{H}]\rangle_\varrho^2}{(\Delta M)^2}.$$

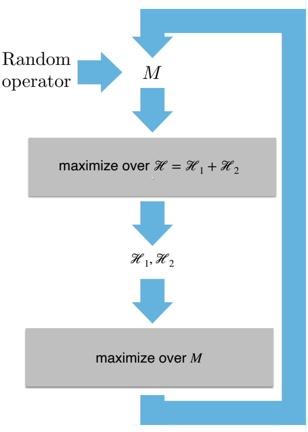
[M. G. Paris, Int. J. Quantum Inform. 2009. Used, e.g., in F. Fröwis, R. Schmied, and N. Gisin, 2015; Used in K. Macieszczak, arXiv:1312.1356; I. Appelaniz *et al.*, NJP 2015.]

The maximum over local Hamiltonians can be obtained as

$$\max_{\text{local } \mathcal{H}} \mathcal{F}_Q[\varrho, \mathcal{H}] = \max_{\text{local } \mathcal{H}} \max_M \frac{\langle i[M, \mathcal{H}]\rangle_\varrho^2}{(\Delta M)^2}.$$

Similar idea for optimizing over the state, rather than over $\mathcal{H}$:
[K. Macieszczak, arXiv:1312.1356; K. Macieszczak, M. Fraas, and R. Demkowicz-Dobrzański, New J. Phys. 16, 113002 (2014);
Tóth and Vértesi, Phys. Rev. Lett. (2018).]

See-saw algorithm for maximizing the precision for given c_1, c_2



Note that $\mathcal{H}_1, \mathcal{H}_2$ fulfill

$$c_n \mathbb{1} \pm \mathcal{H}_n \geq 0.$$

Maximize over $\mathcal{H}$

- We have to maximize
$$\langle i[M, \mathcal{H}] \rangle.$$

- Simple algebra yields
$$\langle i[M, \mathcal{H}] \rangle = \mathrm{Tr}(A_1 \mathcal{H}_1) + \mathrm{Tr}(A_2 \mathcal{H}_2),$$
where
$$A_n = \mathrm{Tr}_{\{1,2\}\setminus n}(i[\varrho, M])$$
are operators acting on a single subsytem.

- Hence, we can maximize $\langle i[M, \mathcal{H}] \rangle$ over $\mathcal{H}_1$ and $\mathcal{H}_2$.

Maximize over $\mathcal{H}$ II

- The optimal $\mathcal{H}_n$ is the one that maximizes $\mathrm{Tr}(A_n\mathcal{H}_n)$ under these constraints. It can straightforwardly be obtained as

$$\mathcal{H}_n^{(\mathrm{opt})} = U_n\tilde{D}_nU_n^\dagger,$$

  where the eigendecompisition of A is given as

$$A_n = U_nD_nU_n^\dagger$$

  and

$$(\tilde{D}_n)_{k,k} = c_n s((D_n)_{k,k}),$$

  where $s(x) = 1$ if $x \geq 0$, and -1 otherwise.

- Clearly, $\mathcal{H}_n^{(\mathrm{opt})}$ has the same eigenvectors as A_n and has only eigenvalues $+c_n$ and $-c_n$.

Maximize over M

- For a state ϱ, the best precision is obtained with the operator given by the symmetric logarithmic derivative

$$M = 2i \sum_{k,l} \frac{\lambda_k - \lambda_l}{\lambda_k + \lambda_l} |k\rangle\langle l| \langle k|A|l\rangle,$$

where

$$\varrho = \sum_k \lambda_k |k\rangle\langle k|.$$

Metrological gain for given c_1, c_2 and for all c_1, c_2

- After several iterations of the two steps above, we obtain the maximal quantum Fisher information over a certain set of Hamiltonians.
- Based on that, we can calculate the quantity

$$g_{c_1,c_2}(\varrho) = \max_{\mathcal{H}_1,\mathcal{H}_2} \frac{\mathcal{F}_Q(\varrho, \mathcal{H}_1 \otimes \mathbb{1} + \mathbb{1} \otimes \mathcal{H}_2)}{\mathcal{F}_Q^{(\mathrm{sep})}(c_1, c_2)},$$

where we assumed that $\mathcal{H}_n$ are constrained with

$$c_n \mathbb{1} \pm \mathcal{H}_n \geq 0.$$

- The separable limit for Hamiltonians of the form $\mathcal{H} = \mathcal{H}_1 \otimes \mathbb{1} + \mathbb{1} \otimes \mathcal{H}_2$ is

$$\mathcal{F}_Q^{(\mathrm{sep})}(\mathcal{H}) = \sum_{n=1,2} [\sigma_{\max}(\mathcal{H}_n) - \sigma_{\min}(\mathcal{H}_n)]^2,$$

which leads to $\mathcal{F}_Q^{(\mathrm{sep})}(c_1, c_2) = 4(c_1^2 + c_2^2)$. It is a constant.

Metrological gain for given c_1, c_2 and for all c_1, c_2 II

- Then, the gain can be expressed as

$$g(\varrho) = \max_{c_2} g_{c_1,c_2}(\varrho),$$

where the optimization is only over c_2, and, without the loss of generality, we set

$$c_1 = 1.$$

- Why do we do it this way? Because direct maximization of

$$g = \frac{\mathcal{F}_Q[\varrho, \mathcal{H}]}{\mathcal{F}_Q^{(\mathrm{sep})}(\mathcal{H})}$$

over the Hamiltonian is difficult, since both the numerator and the denominator must be maximized.

Convergence of the method

The precision cannot get worse with the iteration!

Numerical results

- We remember that the 3×3 isotropic state is useful if

$$p < \frac{25 - \sqrt{177}}{32} \approx 0.3655.$$

- Then, we have the following results for activation.

	Analytic example	Numerics
Ancilla	0.3752	0.3941
Extra copy	0.4164	0.4170

Robustness of the metrological usefulness

$$\varrho(p) = (1-p)\varrho + p\varrho_{\rm noise}$$

- Robustness of entanglement: the maximal p for which $\varrho(p)$ is entangled for any separable $\varrho_{\rm noise}$.
  [Vidal and Tarrach, PRA 59, 141 (1999).]

- Robustness of metrological usefulness: the maximal p for which $\varrho(p)$ outperforms separable state for any separable $\varrho_{\rm noise}$.

Activating by a separable state

- Some PPT state can be activated by a pure product state

$$\varrho_{AB}^{(\mathrm{PPT})} \otimes \varrho_{A'B'}^{(\mathrm{sep})}.$$

- The optimal separable state is a pure product state $\equiv$ 2 pure ancilla qudits.

Outline

All bipartite pure entangled states are useful I

- All entangled bipartite pure states are metrologically useful.

- *Proof.*—For the two-qubit case, see
P. Hyllus, O. Gühne, and A. Smerzi, Phys. Rev. A 82, 012337 (2010).

- General case, pure state with a Schmidt decomposition

$$|\Psi\rangle = \sum_{k=1}^{s} \sigma_k |k\rangle_A |k\rangle_B,$$

where s is the Schmidt number, and the real positive σ_k Schmidt coefficients are in a descending order.

- We define

$$\mathcal{H}_A = \sum_{n=1,3,5,...,\tilde{s}-1} |+\rangle\langle+|_{A,n,n+1} - |-\rangle\langle-|_{A,n,n+1},$$

where $\tilde{s}$ is the largest even number for which $\tilde{s} \leq s$, and

$$|\pm\rangle_{A,n,n+1} = (|n\rangle_A \pm |n+1\rangle_A)/\sqrt{2}.$$

All bipartite pure entangled states are useful II

- We define $\mathcal{H}_B$ in a similar manner.

- We also define the collective Hamiltonian

$$\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathbb{1} + \mathbb{1} \otimes \mathcal{H}_B.$$

  Then, we have $\langle \mathcal{H}_{AB} \rangle_\Psi = 0$.

- Direct calculation yields

$$\mathcal{F}_Q[|\Psi\rangle, \mathcal{H}_{AB}] = 4(\Delta \mathcal{H}_{AB})^2{}_\Psi = 8 \sum_{n=1,3,5,\ldots,\tilde{s}-1} (\sigma_n + \sigma_{n+1})^2,$$

  which is larger than the separable bound, $\mathcal{F}_Q^{(\mathrm{sep})} = 8$, whenever the Schmidt rank is larger than 1.

All bipartite pure entangled states are useful III

- For even s, this can be seen noting that

$$\mathcal{F}_Q[|\Psi\rangle, \mathcal{H}_{AB}] > 8\sum_{n=1}^{s} \sigma_n^2$$

holds, where we took into account that $\sigma_n > 0$ for $n = 1, 2, 3, ...,$ and $\sum_{n=1}^{s} \sigma_n^2 = 1$.

- For odd s, we need that

$$\mathcal{F}_Q[|\Psi\rangle, \mathcal{H}_{AB}] \geq 8\left(\sum_{n=1}^{s-1} \sigma_n^2 + 2\sigma_1\sigma_2\right) > 8\sum_{n=1}^{s} \sigma_n^2$$

holds, where we used that $\sigma_1\sigma_2 > \sigma_s^2$. $\square$

Infinite number of copies

- In the infinite copy limit, all bipartite pure entangled states are maximally useful.
- For the proof, see the
  [Supplemental Material of Tóth, Vértesi, Horodecki, Horodecki, PRL 2020.]

Summary

- Some entangled quantum states that are not more useful for metrology than separable states can still be made more useful, if an ancilla or an additional copy is added.
- We have shown a general method to get the optimal local Hamiltonian for a quantum state.
- All bipartite pure entangled states are useful metrologically. If infinite number of copies are given, they are maximally useful.

See:

Géza Tóth, Tamás Vértesi, Paweł Horodecki, Ryszard Horodecki,

Activating hidden metrological usefulness,

Phys. Rev. Lett. 125, 020402 (2020). (open access)

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