Activating hidden metrological usefulness

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Photos

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What are entangled states useful for?

- Entanglement is needed for beating the shot-noise limit in quantum metrology.
- Not all entangled states are more useful than separable states.
- Question:
  - Can a quantum state become useful metrologically, if an ancilla or a second copy is added?
  - How to find the local Hamiltonian, for which a quantum state is the most useful compared to separable states?

Metrological usefulness

- Metrological gain optimized over all local Hamiltonians

$$g(\varrho) = \max_{\text{local}\mathcal{H}} \frac{\mathcal{F}_Q[\varrho, \mathcal{H}]}{\mathcal{F}_Q^{(\text{sep})}(\mathcal{H})} \quad \begin{array}{l} \leftarrow \text{metrological performance of } \varrho \\ \leftarrow \text{best metrological performance of} \\ \quad \text{separable states} \end{array}$$

- A state ϱ is useful and also entangled if $g(\varrho) > 1$.
- We would like to determine g.
- It is a very important problem in metrology!
- It is difficult, since $\mathcal{H}$ appears in both the numerator and the denominator!

An element of the algorithm:

Maximization of $\mathcal{F}_Q[\varrho, \mathcal{H}]$ over $\mathcal{H}$ is difficult: it is convex in $\mathcal{H}$.

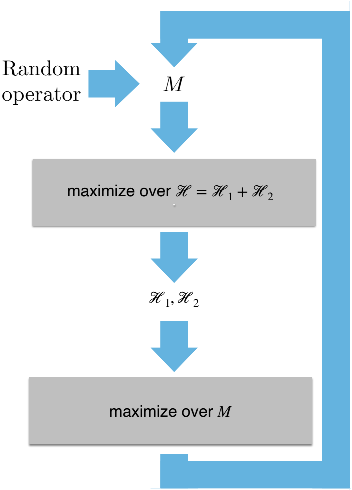
The maximum over local Hamiltonians can be obtained as

$$\max_{\text{local } \mathcal{H}} \mathcal{F}_Q[\varrho, \mathcal{H}] = \max_{\text{local } \mathcal{H}} \max_{M} \frac{\langle i[M, \mathcal{H}] \rangle_\varrho^2}{(\Delta M)^2}.$$

Similar idea for optimizing over the state, rather than over $\mathcal{H}$:

[K. Macieszczak, arXiv:1312.1356; K. Macieszczak, M. Fraas, and R. Demkowicz-Dobrzański, New J. Phys. 16, 113002 (2014);
Tóth and Vértesi, Phys. Rev. Lett. (2018).]

See-saw algorithm



The precision cannot get worse with the iteration!

Note that $\mathcal{H}_1, \mathcal{H}_2$ fulfill

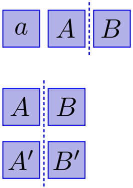
$$c_n \mathbb{1} \pm \mathcal{H}_n \geq 0.$$

Numerical results for isotropic state

- The 3×3 isotropic state is useful if

$$p < \frac{25 - \sqrt{177}}{32} \approx 0.3655.$$

- Then, we have the following results for activation.



	Analytic example	Numerics
Ancilla	0.3752	0.3941
Second copy	0.4164	0.4170

Pure states

- All bipartite pure entangled states are useful.
- In the infinite copy limit, all bipartite pure entangled states are maximally useful.

Summary

See:

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