

**Loophole free Bell inequality test
(Supplementary material for the Lecture of the
Quantum Information class of the Master in
Quantum Science and Technology)**

Géza Tóth

Theoretical Physics, University of the Basque Country (UPV/EHU), Bilbao, Spain
Donostia International Physics Center (DIPC), San Sebastián, Spain
IKERBASQUE, Basque Foundation for Science, Bilbao, Spain
Wigner Research Centre for Physics, Budapest, Hungary

UPV/EHU, Leioa
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1 Motivation

- Why is photonic implementation of quantum information processing interesting?

2 Bell inequalities

- CHSH Bell's inequality

3 The experiment

- Basic idea

Why is photonic implementation of quantum information processing interesting?

- It is important to prove that a Bell inequality violated, since this makes it sure that hidden variable models cannot be used to describe the world.
- We have to exclude loopholes, such as detection loophole and locality loophole to be fully sure.
- We will now review the paper

W. Rosenfeld, D. Burchardt, R. Garthoff, K. Redeker, N. Ortegel, M. Rau, and H. Weinfurter, Event-Ready Bell Test Using Entangled Atoms Simultaneously Closing Detection and Locality Loopholes, Phys. Rev. Lett. 119, 010402 (2017).

[\[link to Physical Review Letters, press 'pdf'\]](#).

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CHSH Bell's inequality

- Let us imagine that we have a bipartite system with parties 1 and 2. We measure X_1 and Y_1 on party 1, and X_2 and Y_2 on party 2. All X_k, Y_k measurements have an outcome $+1$ or -1 .
- The CHSH Bell inequality has the following form

$$X_1 X_2 + X_1 Y_2 + Y_1 X_2 - Y_1 Y_2 \leq 2.$$

It is a bound for Local Hidden Variable (LHV) models. These models assume that the measurement results exist locally before the measurement, and we just read out the one we choose.

- In practice, the bound can be obtained by substituting $+1$ and -1 into X_k and Y_k , and looking for the maximum:

$$\text{Bound} = \max_{X_k, Y_k \in \{+1, -1\}} X_1 X_2 + X_1 Y_2 + Y_1 X_2 - Y_1 Y_2$$

Thus we we obtain $\text{Bound} = 2$.

CHSH Bell's inequality II

- Let us write now the CHSH inequality with operators

$$\langle \sigma_x^{(1)} \sigma_x^{(2)} \rangle + \langle \sigma_x^{(1)} \sigma_y^{(2)} \rangle + \langle \sigma_y^{(1)} \sigma_x^{(2)} \rangle - \langle \sigma_y^{(1)} \sigma_y^{(2)} \rangle \leq 2.$$

- There are quantum states that can reach

$$\langle \sigma_x^{(1)} \sigma_x^{(2)} \rangle + \langle \sigma_x^{(1)} \sigma_y^{(2)} \rangle + \langle \sigma_y^{(1)} \sigma_x^{(2)} \rangle - \langle \sigma_y^{(1)} \sigma_y^{(2)} \rangle = 2\sqrt{2},$$

thus they violate the CHSH Bell inequality. The state giving $2\sqrt{2}$ is a maximally entangled state $(|00\rangle + |11\rangle)/\sqrt{2}$, apart from local operations.

Loopholes

- Loopholes, in principle, could make it possible that Bell inequalities seem to be violated, but in reality they are not violated.
- The first loophole is the **detection loophole**. Many detectors cannot detect all particles. Thus, part of the experiments are unnoticed.
- This is the largest problem with photonic experiments, while with atomic qubits or ions it is not a problem.
- The other loophole is the **locality loophole**. If the two subsystems are too close to each other, one subsystem can "know" what operator is measured on the other. This way, the experimental results could be obtained even with local hidden variable models. This is problematic with trapped ions stored close to each other, or atoms in the same trap.

Outline

1 Motivation

- Why is photonic implementation of quantum information processing interesting?

2 Bell inequalities

- CHSH Bell's inequality

3 The experiment

- Basic idea

Lophole-free test

- Two Rb (Rubidium) atoms are the qubits, they are far away from each other.
- Each emits a photon and measuring these photons they entangle the atoms.
- Then, they measure the two atoms independently in a space-like separated manner. At the first atom a circuit decides locally what to measure, X or Y, and the second atoms is very far, it cannot know about it.

CHSH inequality

- In the paper, they write the CHSH inequality little differently. In the first subsystem, they measure $\sigma_\alpha \sigma_{\alpha'}$. On the other subsystem, they measure $\sigma_\beta \sigma_{\beta'}$.

We consider the simplest situation of an event-ready Bell test, where two separate observers are told—according to a heralding signal—to report the result of two-outcome measurements $A, B \in \{\uparrow, \downarrow\}$ performed on each side (an example are measurements on spin- $\frac{1}{2}$ particles). For a test of local realism the two observers choose their measurement directions from two possibilities $a \in \{\alpha, \alpha'\}$ and $b \in \{\beta, \beta'\}$ and afterwards compare their results. For this situation Clauser, Horne, Shimony, and Holt (CHSH) put Bell's inequality in an experimentally friendly form [22]:

$$S = |\langle \sigma_\alpha \sigma_\beta \rangle + \langle \sigma_\alpha \sigma_{\beta'} \rangle| + |\langle \sigma_{\alpha'} \sigma_\beta \rangle - \langle \sigma_{\alpha'} \sigma_{\beta'} \rangle| \leq 2, \quad (1)$$

with correlators $\langle \sigma_a \sigma_b \rangle = (1/N_{a,b})(N_{a,b}^{\uparrow\uparrow} + N_{a,b}^{\downarrow\downarrow} - N_{a,b}^{\uparrow\downarrow} - N_{a,b}^{\downarrow\uparrow})$. Here $N_{a,b}^{A,B}$ denote the number of events with the respective outcomes A, B for measurement directions a, b and $N_{a,b}$ is the total number of events of the

- The added absolute $|\dots|$ value does not change the main point: the bound comes from substituting $+1$ and -1 to the measured quantities.

Qubits are stored in atoms

- The two qubits are stored in Rb atoms, 398m from each other:

In our case the two observer stations are independently operated setups (trap 1 and trap 2) that are equipped with their own laser and control systems. Their separation of 398 m (Fig. 1) makes 1328 ns available to warrant spacelike separation of the measurements. On each side we store a single ^{87}Rb atom in an optical dipole trap. The employed internal spin states ($|\uparrow\rangle_z$ and $|\downarrow\rangle_z$) are the Zeeman states $|m_F = +1\rangle$ and $|m_F = -1\rangle$ of the ground level $5^2S_{1/2}$, $F = 1$ [Fig. 2(a)]. Entanglement of the atoms is generated by first

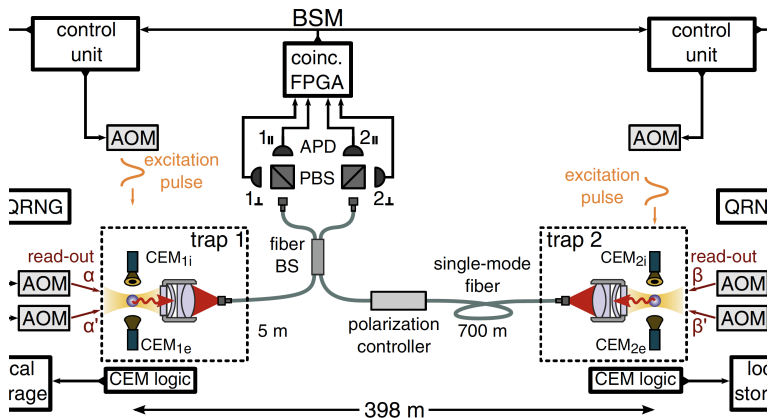
Entanglement generation between the atomic qubits

- Each atom is entangled with a photon. Then the two photons are measured in the Bell basis, which entangles the atoms with each other.

[Fig. 2(a)]. Entanglement of the atoms is generated by first entangling the spin of each atom with the polarization of a single emitted photon [11]. The photons are guided to an interferometric Bell state measurement (BSM) setup (Fig. 2), located close to trap 1. It consists of a fiber beam splitter (BS) followed by polarizing beam splitters (PBS) in each of the output ports, where detection of photons is performed by four avalanche photodiodes (APDs). This setup allows us to distinguish two maximally entangled photon states. Thereby a two-photon coincidence in particular detector combinations (see Sec. I. B of the Supplemental Material [23], which includes Refs. [24–30]) heralds the projection of the atoms onto one of the states $|\Psi^\pm\rangle = (1/\sqrt{2})(|\uparrow\rangle_x|\downarrow\rangle_x \pm |\downarrow\rangle_x|\uparrow\rangle_x)$ [13], where $|\uparrow\rangle_x = (1/\sqrt{2})(|\uparrow\rangle_z + |\downarrow\rangle_z)$ and $|\downarrow\rangle_x = (i/\sqrt{2})(|\uparrow\rangle_z - |\downarrow\rangle_z)$.

Entanglement generation between the atomic qubits II

- The relevant part is the central part of Fig. 2:



How to measure the Bell inequality on the two qubits

- Atoms in a given state are ionized, the orthogonal state remains unaffected. The ionization fragments (Rb^+ and electrons) are detected.

For the analysis of the atomic state a state-selective ionization is employed where the measurement direction $\gamma \in \{\alpha, \alpha', \beta, \beta'\}$ is determined by the polarization of a readout laser at 795 nm exciting the atom to the $5^2P_{1/2}$, $F' = 1$ level from where it is ionized by an additional laser at 473 nm [Fig. 2(b)]. In particular, we ionize the state $|\uparrow\rangle_\gamma = \sin(\gamma/2)|\uparrow\rangle_x - \cos(\gamma/2)|\downarrow\rangle_x$ using linear polarization at an angle $\gamma/2$ relative to the horizontal. The state $|\downarrow\rangle_\gamma = \cos(\gamma/2)|\uparrow\rangle_x + \sin(\gamma/2)|\downarrow\rangle_x$ remains unaffected. The resulting $^{87}\text{Rb}^+$ ion and electron are accelerated by an electric field to two channel electron multipliers (CEMs) placed in 8 mm distance from the trapping region. The ionization fragments are detected with high efficiencies $\eta_i = 0.90\dots 0.94$ (ions), $\eta_e = 0.75\dots 0.90$ (electrons); the

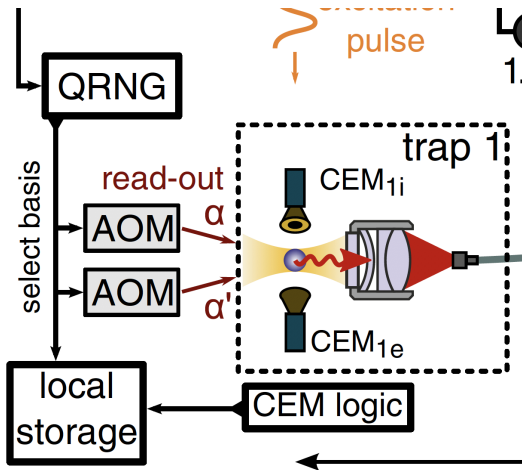
How to measure the Bell inequality on the two qubits II

- The detection efficiency is very high ($0.98 = 98\%$) which is important to exclude the detection loophole.

efficiencies are slightly different for the two labs and also vary between different measurement runs. We assign detection of at least one of the fragments to the atomic state $|\uparrow\rangle_\gamma$, providing a total detection efficiency of ≥ 0.98 [36,37], while detection of no fragment is assigned to the state $|\downarrow\rangle_\gamma$. Note that in the event-ready scheme an imperfect detection efficiency does only affect the fidelity of the measurement process.

How to measure the Bell inequality on the two qubits II

- At each qubit, there is the following unit, which can read out the qubit.



Space-like separated setup

- The two-qubits are far from each other. The process of (1) selecting which observable to measure, and (2) measuring it, is needs smaller time than the time needed for light to travel from one qubit to the other.
- Thus, they can exclude the locality loophole.

Conclusions

- We discussed a loophole free Bell inequality experiment described in the paper:

W. Rosenfeld, D. Burchardt, R. Garthoff, K. Redeker, N. Ortegel, M. Rau, and H. Weinfurter, Event-Ready Bell Test Using Entangled Atoms Simultaneously Closing Detection and Locality Loopholes, Phys. Rev. Lett. 119, 010402 (2017).

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- See the video with the presentation of Harald Weinfurter (Max Planck Institute for Quantum Optics, Munich) at

[\[link to youtube video\]](#).

THANK YOU FOR YOUR ATTENTION!