Controlled collisions for multi-particle entanglement of optically trapped atoms — we review a paper (Lecture of the Quantum Information class of the Master in Quantum Science and Technology)

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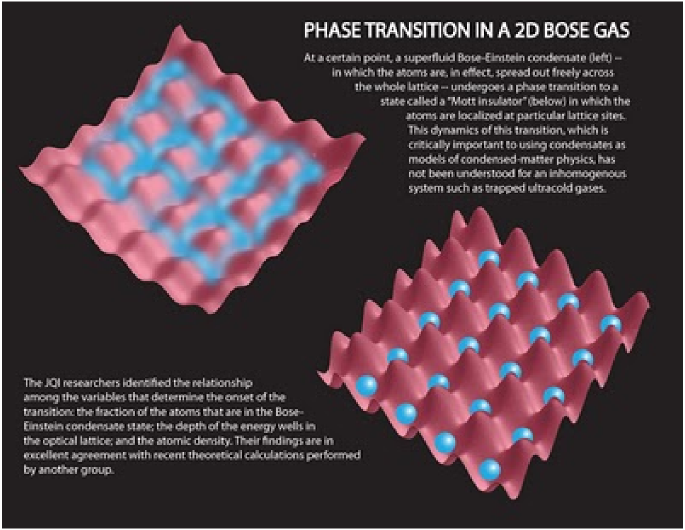
17 Febrtuary 2022

Optical lattices of cold atoms



Superfluid-Mott insulator phase transition, MPQ, Munich.
[Greiner, Mandel, Esslinger, Hänsch & Bloch, Nature 2002]

Optical lattices of cold atoms II

- Hamiltonian: Bose-Hubbard model for two-state atoms:

$$
\begin{aligned}
H = J_a \sum_k a_k a_{k+1}^\dagger + a_k^\dagger a_{k+1} \\
+ J_b \sum_k b_k b_{k+1}^\dagger + b_k^\dagger b_{k+1} \\
+ \sum_k U_a n_{a,k}(n_{a,k} - 1) \\
+ U_b n_{b,k}(n_{b,k} - 1) + U_{ab} n_{a,k} n_{b,k}.
\end{aligned}
$$

- Tunneling between sites for species *a* and *b*, self-interaction for species *a* and *b*, and interaction between the two species.

Trapping atoms in an optical lattices

- The idea of trapping with light is that they can trap the atoms such that the atoms "feel a force" towards areas with a high light intensity.

- This happens when they use red detuning, that is, they use a frequency smaller than the energy difference between the ground state and the excited state of a two-state atom. (It can also happen that they feel a force towards low areas with a low light intensity, when they use blue detuning.)

- This is the basis of optical dipole traps for neutral atoms.

- See Eqs. (15) and (16), and Fig. 1 in

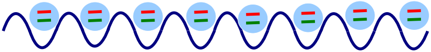
  R. Grimm, M Weidemüller, and Y. B. Ovchinnikov,
  Advances in Atomic, Molecular and Optical Physics Vol. 42, 95-170 (2000);
  link: https://arxiv.org/abs/physics/9902072.

Controlled collisions I

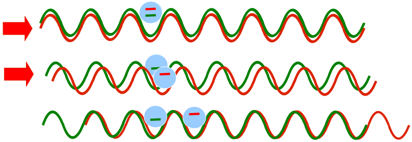
- We will now review the paper

  O. Mandel, M. Greiner, A. Widera, T. Rom, Th. W. Hänsch and I. Bloch,
  Controlled collisions for multi-particle entanglement of optically trapped atoms,
  Nature 425, 937 (2003).

- In the experiment described in the paper, they use two potentials for two atomic states



- Atoms in the two basis states can be trapped by different potentials



- An atom can be delocalized by several lattices sites.

Controlled collisions II

- Two-particle example. They start from a $|00\rangle_z$ state.

lattice sites. To illustrate this, let us consider the case of two neighbouring atoms, initially in state $|\Psi\rangle = |0\rangle_j|0\rangle_{j+1}$ placed on the jth and $(j+1)$th lattice site of the periodic potential in the spin-state $|0\rangle$. First, both atoms are brought into a superposition of two

- They create an $|11\rangle_x$ state by a $\pi/2$ rotation around the y-axis.

state $|0\rangle$. First, both atoms are brought into a superposition of two internal states $|0\rangle$ and $|1\rangle$, using a $\pi/2$ pulse such that $|\Psi\rangle = (|0\rangle_j + |1\rangle_j)(|0\rangle_{j+1} + |1\rangle_{j+1})/2$. Then, a spin-dependent transport[18]

- They move the optical lattice trapping atoms in state $|1\rangle$ with respect to the lattice trapping atoms in state $|0\rangle$

$(|0\rangle_j + |1\rangle_j)(|0\rangle_{j+1} + |1\rangle_{j+1})/2$. Then, a spin-dependent transport[18] splits the spatial wave packet of each atom such that the wave packet of the atom in state $|0\rangle$ moves to the left, whereas the wave packet of the atom in state $|1\rangle$ moves to the right. The two wave packets are separated by a distance $\Delta x = \lambda/2$, such that now $|\Psi\rangle = (|0\rangle_j|0\rangle_{j+1} + |0\rangle_j|1\rangle_{j+2} + |1\rangle_{j+1}|0\rangle_{j+1} + |1\rangle_{j+1}|1\rangle_{j+2})/2$, where in the notation atoms in state $|0\rangle$ have retained their original lattice site index and λ is the wavelength of the laser forming the optical periodic potential. The collisional interaction between the atoms[5,12,19] over a

- Note the term $|1\rangle_{j+1}|0\rangle_{j+1}$, which corresponds to the case that the two atoms are at the same site.

Controlled collisions III

- Atoms on the same site interact with each other, due to that the term $|1\rangle_{j+1}|0\rangle_{j+1}$, picks up a phase

potential. The collisional interaction between the atoms[5,12,19] over a time t_{hold} will lead to a distinct phase shift $\varphi = U_{01} t_{\mathrm{hold}}/\hbar$, when

both atoms occupy the same lattice site $j+1$ resulting in: $|\Psi\rangle = (|0\rangle_j|0\rangle_{j+1} + |0\rangle_j|1\rangle_{j+2} + \mathrm{e}^{-i\varphi}|1\rangle_{j+1}|0\rangle_{j+1} + |1\rangle_{j+1}|1\rangle_{j+2})/2$. Here U_{01}

- This way they realize a two-qubit unitary gate

$$U = diag(1, 1, \exp(-i\phi), 1) \equiv \exp\left(-i\frac{\mathbb{1} - \sigma_z}{2} \otimes \frac{\mathbb{1} + \sigma_z}{2}\phi\right).$$

- After another $\pi/2$ pulse (rotating back) we obtain

proposed[11] for generating a state-dependent phase shift φ. The final many-body state after bringing the atoms back to their original site and applying a last $\pi/2$ pulse can be expressed as $|\Psi\rangle = \frac{1+e^{-i\varphi}}{2}|1\rangle_j|1\rangle_{j+1} + \frac{1-e^{-i\varphi}}{2}|\mathrm{BELL}\rangle$. Here $|\mathrm{BELL}\rangle$ denotes the Bell-like state corresponding to $(|0\rangle_j(|0\rangle_{j+1} - |1\rangle_{j+1}) + |1\rangle_j(|0\rangle_{j+1} + |1\rangle_{j+1}))/2$.

Thus, for $\phi = \pi$, we get the Bell state. For $\phi = 2\pi$, we obtain again the initial state.

Controlled collisions IV

- For three particles, we can produce a Grenberger-Horne-Zielinger (GHZ) state

This scheme can be generalized when more than two particles are placed next to each other, starting from a Mott insulating state of matter[9,10]. In such a Mott insulating state, atoms are localized to lattice sites, with a fixed number of atoms per site. For three particles for example, one can show that if $\varphi = (2n + 1)\pi$ (with n being an integer), so-called maximally entangled Greenberger–Horne–Zeilinger (GHZ) states[20] are realized. For a string of $N > 3$ atoms,

- For more three particles, we can produce a so-called cluster state. It is a highly entangled state

Zeilinger (GHZ) states[20] are realized. For a string of $N > 3$ atoms,
where each atom interacts with its left- and right-hand neighbour (see Fig. 1), the entire string of atoms can be entangled to form so-called cluster states in a single operational step[5,6]. The controlled

Controlled collisions V

- The dynamics for N particles is

$$U = \exp\left(-i\sum_{n=1}^{N-1}\frac{\mathbb{1}-\sigma_z^{(n)}}{2}\otimes\frac{\mathbb{1}+\sigma_z^{(n+1)}}{2}\phi\right).$$

Apart from local unitaries, this is an Ising dynamics.

- This can be considered as two-qubit phase gates acting in paralelel:

called cluster states in a single operational step[5,6]. The controlled interactions described above can be viewed as being equivalent to an ensemble of quantum gates acting in parallel[3,5].

$$U = U_{12}U_{23}...U_{(N-1)N},$$

where the two-qubit gate is

$$U_{n(n+1)} = \exp\left(-i\frac{\mathbb{1}-\sigma_z^{(n)}}{2}\otimes\frac{\mathbb{1}+\sigma_z^{(n+1)}}{2}\phi\right).$$