Permutationally invariant tomography of symmetric Dicke states

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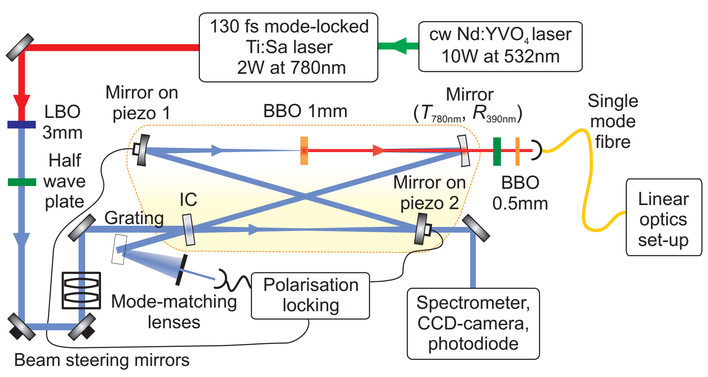
Abstract

Multi-partite entangled quantum states play a crucial role for quantum information processing. Tóth et al. [1] developed a method where the measurement effort under the restriction of permutational invariance (PI) scales only polynomially. This is of great importance since many prominent quantum states like, for example, GHZ states, W states or symmetric Dicke states are permutationally invariant. Our state reconstruction algorithm is tailored to the permutational invariant nature of the problem leading to a drastic reduction of its dimensionality. It employs convex optimization making it superior to other numerical optimization methods in terms of speed, accuracy and control [2].
Here, we present experimental results of the tomographic analysis of a photonic six qubit symmetric Dicke state, as obtained from parametric down-conversion in a UV-femtosecond enhancement cavity [3,4]. For such systems full tomography is extremely challenging due to an exceedingly high number of 3^6=729 measurement settings and low count rates. In comparison permutationally invariant tomography needs only 28 measurement settings. Our experiment shows that a tomographic reconstruction of states of arbitrary qubit number is in principle feasible, when restricting to the permutationally invariant subspace.

New SPDC Pump Source

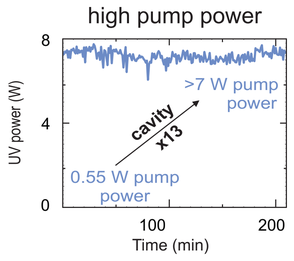
• fs enhancement cavity in the UV

- schematic

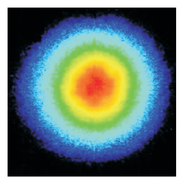


- characterization of the cavity

high pump power

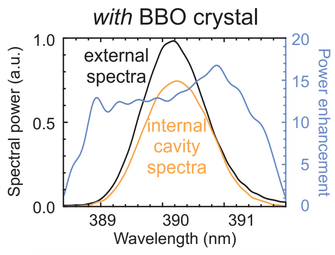


pump beam profile



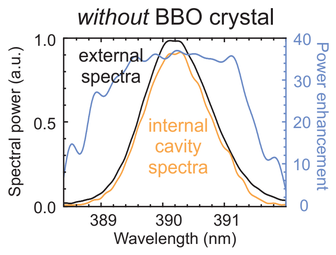
M^2 = 1.15±0.03

with BBO crystal



power enhancement x13

without BBO crystal

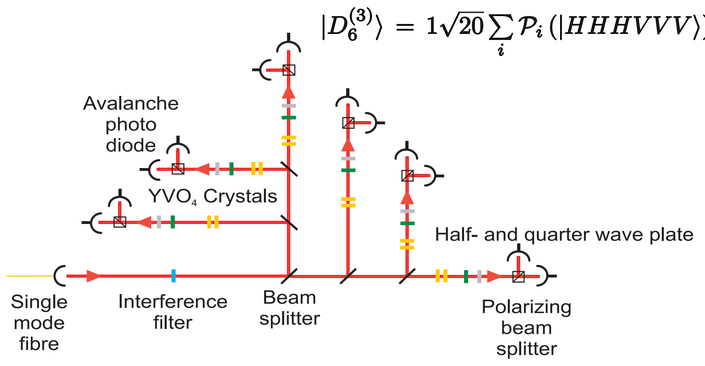


power enhancement x37

• Linear optical setup

- six photon Dicke setup to generate the state $|D_6^{(3)}\rangle$

$$|D_6^{(3)}\rangle = 1\sqrt{20}\sum_i \mathcal{P}_i\,(|HHHVVV\rangle)$$

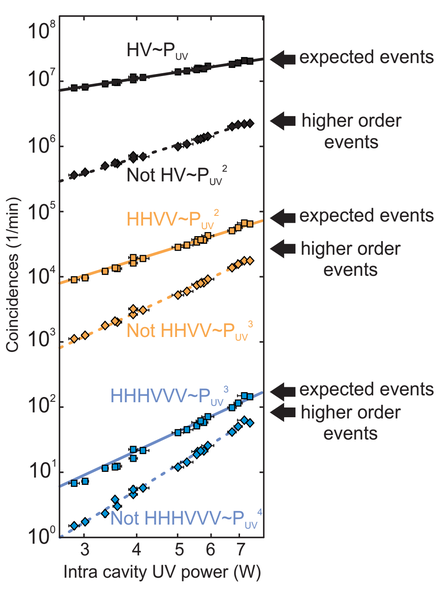


input state from the photon source is split up into spacial modes
success rate of the setup is 1.3 % for six photons
conditional detection scheme is applied

• Higher orders

- SPDC source delivers all orders at the same time

$$|\Psi_{SPDC}\rangle \propto e^{ic(\hat{a}_H^\dagger \hat{a}_V^\dagger)}|vac\rangle \propto |vac\rangle + \underbrace{ic(\hat{a}_H^\dagger \hat{a}_V^\dagger)|vac\rangle}_{1st\ order} - \underbrace{\frac{c^2}{2}(\hat{a}_H^\dagger \hat{a}_V^\dagger)^2|vac\rangle}_{2nd\ order} - \underbrace{\frac{ic^3}{6}(\hat{a}_H^\dagger \hat{a}_V^\dagger)^3|vac\rangle}_{3rd\ order} + \ldots$$



investigation of six-partite entangled symmetric Dicke states requires *reasonable* count rates

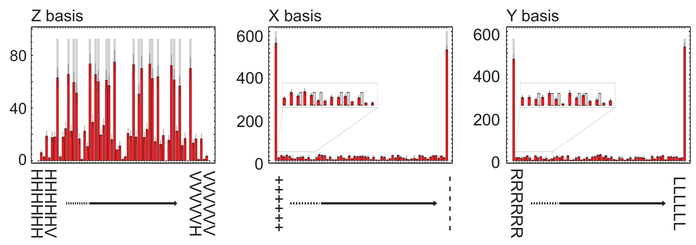
high pump power of up to 8.6 W leads to a stong increase in higher order events but *also* to an increase in colored noise

Quantum State Analysis

• Symmetric six photon Dicke state

- correlation measurements

six-fold coincides (counts in 513 min)



- fidelity F was determined by measurement of 21 settings
threshold for six-partite entanglement is F > 60%

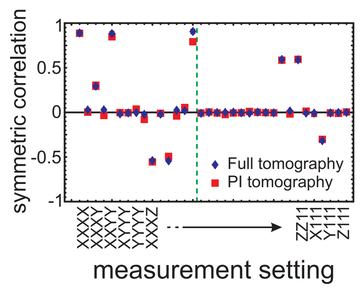
Fidelity F = 65.4 ± 2.4%

• Permutationally invariant tomography

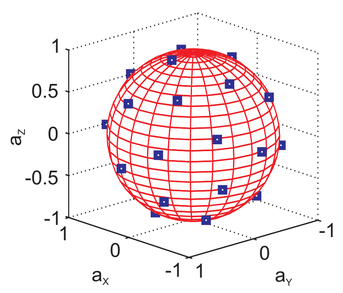
- efficient tomography: number of measurement settings scales only ***qudratically*** with the number of qubits
- applicable for permutationally invariant states like GHZ, W or symmetric Dicke states
- three setting test measurement $X^{\otimes N}$, $Y^{\otimes N}$ and $Z^{\otimes N}$ to determine the overlap with the symmetric subspace
- for each measurement the same local setting is applied to all qubits

• Proof of principle experiment with $|D_4^{(2)}\rangle$

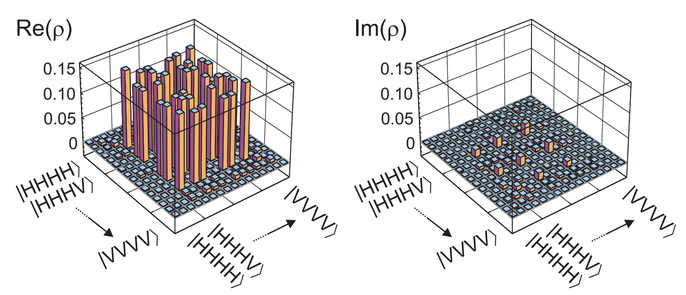
- symmetric correlations from full and PI tomography



- measurement directions for PI tomography

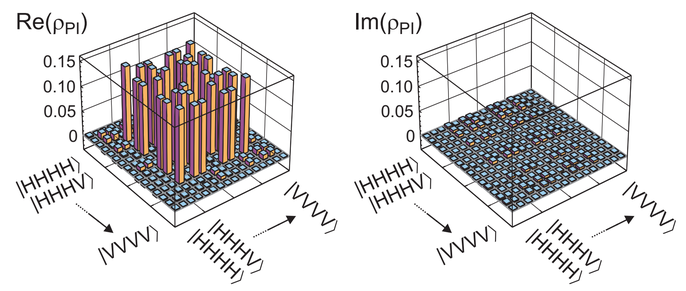


full tomography of $|D_4^{(2)}\rangle$ with 81 settings



Fidelity F = 86.9%

PI tomography of $|D_4^{(2)}\rangle$ with 15 settings



Fidelity F = 85.0%

- Overlap of full and permutationally invariant tomography

Fidelity F = 94.6%

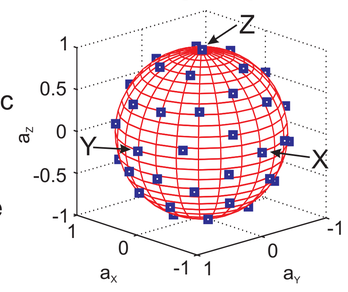
Six Qubit Tomography

• Experimental facts

- Four different UV pump levels from 3.7 W up to 8.6 W
- Mean six fold count rate up to 18.6/min
- Average counts per basis setting >330
- 28 basis settings versus 729 settings for full tomography

- Overlap with the symmetric subspace can be determined ***a priori*** from ***only three*** basis measurements
  - ➔ overlap with the symmetric subspace >88.3%
  - ➔ permutationally invariant tomography makes sense

- measurement directions for PI tomography



• Efficient state reconstruction algorithm

- Reduction of the number of degrees of freedom due to symmetry

Full tomography: $4^N = 4096$
symmetry
PI tomography: $\binom{N+3}{N} = 84$

- Change to spin basis

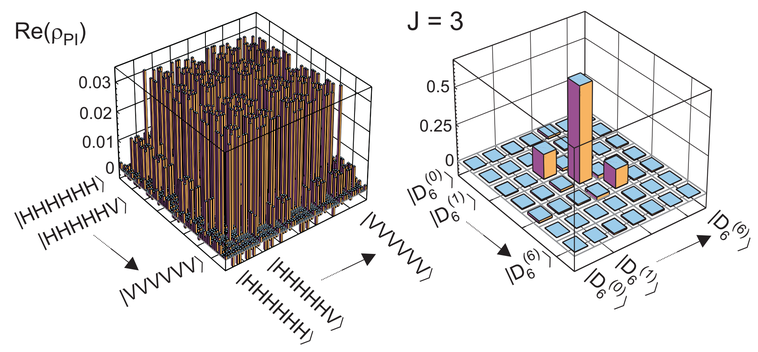
$$\rho_{PI} = \bigoplus_{j=j_{min}}^{N/2} \frac{\mathbb{1}}{\dim(\mathcal{K}_j)} \otimes \rho_j\,\rho_j$$

$$\rho_{PI} = \begin{pmatrix} \tilde{\rho}_{N/2} & & & \\ & \mathcal{K}_j \otimes \mathcal{H}_j & & \\ & & \tilde{\rho}_j & 0 \\ & & 0 & \tilde{\rho}_j \\ & & & & \tilde{\rho}_0 \end{pmatrix}$$

- Convex optimization
- Maximum likelihood and least squares methods
- State reconstruction of a 20 qubit state in 10 minuntes on a standard desktop PC

• Experimental state after fitting

PI tomography of $|D_6^{(3)}\rangle$ with 28 settings at 3.7 W and a rate of 2.3/min



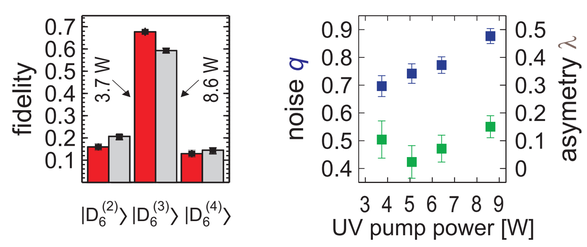
- If ρ_{PI} is entangled so is ρ ➔ six-partite entanglement

Fidelity F = 67.8 ± 1.8%

- Analysis of higher order noise originating from eight-photon events

$$\rho_{exp} = (1\text{-}q)\rho_{|D_6^{(3)}\rangle} + q\rho_{noise}$$

$$\rho_{noise} = \frac{4}{7}\rho_{|D_6^{(3)}\rangle} + \frac{3}{14}[(1+\lambda)\rho_{|D_6^{(2)}\rangle} + (1\text{-}\lambda)\rho_{|D_6^{(4)}\rangle}]$$



References

[1] G. Tóth et al., PRL **105**, 250403 (2010)
[2] T. Moroder et al., arXiv:1205.4941v1, (2012)
[3] W. Wieczorek et al., PRL **103**, 020504 (2009)
[4] R. Krischek et al., Nature Photonics **4**, 170 (2010)

Outlook

- Comparison of permutationally invariant tomography and compressed sensing for symmetric low rank states
- Check applicability of permutationally invariant tomography for process tomography
- Full tomography of a six qubit state