Generation of macroscopic singlet states in atomic ensembles

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Outline

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Motivation

- In many quantum control experiments the qubits cannot be individually accessed. We still would like to create and detect entanglement.

- In such systems, entanglement creation and detection is possible through spin squeezing. We would like to use the ideas behind the spin squeezing approach such that
  - We could create and detect entanglement in systems of particles with arbitrarily large spin
  - We could engineer quantum states other than the classical spin squeezed state with a large spin, that is, unpolarized states.
  - We would also like to generalize the Gaussian approach for describing the dynamics leading to such states.

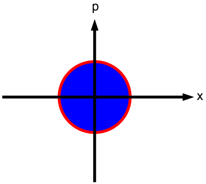
Outline

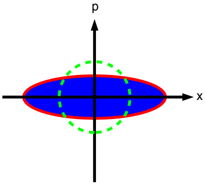
From squeezing to spin squeezing

- The variances of the two quadrature components are bounded

$$(\Delta x)^2(\Delta p)^2 \geq const.$$

- Coherent states saturate the inequality.
- Squeezed states are the states for which one of the quadrature components have a smaller variance than for a coherent state.





- Can one use similar ideas for spin systems?

Spin squeezing

Definition

The variances of the angular momentum components are bounded

$$(\Delta J_x)^2(\Delta J_y)^2 \geq \frac{1}{4}|\langle J_z \rangle|^2,$$

where the mean spin points to the z direction. If $(\Delta J_x)^2$ is smaller than the standard quantum limit $\frac{|\langle Jz \rangle|}{2}$ then the state is called spin squeezed.

- In practice this means that the angular momentum of the state has a small variance in one direction, while in an orthogonal direction the angular momentum is large.

  [M. Kitagawa and M. Ueda, PRA **47**, 5138 (1993).]

Entanglement

Definition

Fully separable states are states that can be written in the form

$$\rho = \sum_l p_l \rho_l^{(1)} \otimes \rho_l^{(2)} \otimes ... \otimes \rho_l^{(N)},$$

where $\sum_l p_l = 1$ and $p_l > 0$.

Definition

A state is entangled if it is not separable.

- Note that one could also look for other type of entanglement in many-particle systems, e.g., entanglement in the two-qubit reduced density matrix.

The standard spin-squeezing criterion

Definition

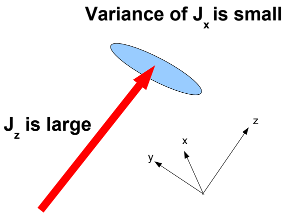
The spin squeezing criterion for entanglement detection is

$$\frac{(\Delta J_x)^2}{\langle J_y\rangle^2 + \langle J_z\rangle^2} \geq \frac{1}{N}.$$

If it is violated then the state is entangled.

[A. Sørensen, L.M. Duan, J.I. Cirac, P. Zoller, Nature **409**, 63 (2001).]

- Note that this criterion is for spin-1/2 particles.
- States violating it are like this:



A generalized spin squeezing entanglement criterion

Separable states of N spin-j particles must fulfill

$$\xi_s^2 := (\Delta J_x)^2 + (\Delta J_y)^2 + (\Delta J_z)^2 \geq Nj.$$

It is maximally violated by a many-body singlet, e.g., the ground state of an anti-ferromagnetic Heisenberg chain.

[GT, PRA **69**, 052327 (2004);GT, C. Knapp, O. Gühne, and H.J. Briegel, PRL **99**, 250405 (2007).]

- For such a state

$$\langle J_k^m \rangle = 0.$$

- $N\xi_s^2$ gives an upper bound on the number of unentangled spins.

Many-body singlet states

Many-body singlet states have been studied a lot in condensed matter physics and quantum information science. They can be created typically in Heisenberg lattices.

- The permutationally invariant singlet state we consider here is the $T = 0$ thermal ground state of

$$H = \frac{1}{N}(J_x^2 + J_y^2 + J_z^2) \text{ or } H = \frac{1}{N}(J_x^2 + J_y^2),$$

  latter for the qubit case being the Lipkin-Meshkov-Glick model. The realization of such states is difficult, since the Hamiltonian is essentially the sum of two-body interactions connecting *all* spin pairs.
- For the qubit case, the bipartite entanglement of such a state is known.
- Surprisingly, this state appears even in quantum gravity calculations of black hole entropy. [E.R. Livine, and D.R. Terno, Phys. Rev. A. **72**, 022307 (2005).]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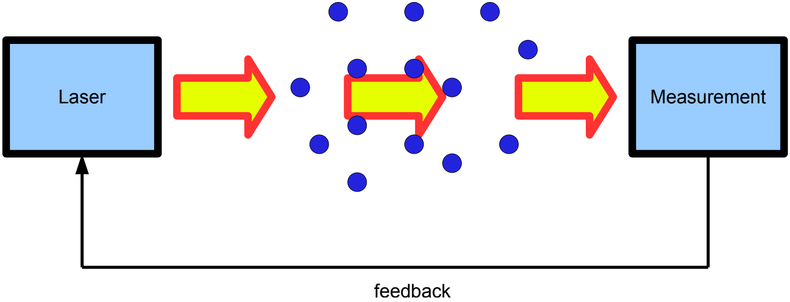
Outline

The physical system: atoms + light

- We consider atoms interacting with light.
- The light is then measured and the atoms are projected into an entangled state.



Quantum non-demolition measurement (QND) of the ensemble

The steps the the QND measurement of J_k :

- 1. Set the light to
$$\langle \mathbf{S} \rangle = (S_0, 0, 0).$$
- 2. The atoms interact with the light for time t
$$H = \Omega J_k S_z$$
- 3. Measurement of S_y.

- The most obvious effect of such a measurement is the decrease of $(\Delta J_k)^2$.
- The timescale of the dynamics, for $J := Nj$, is
$$t \sim \tau := \frac{1}{\Omega \sqrt{S_0 J}}.$$

The proposed protocol

1. Initial state
   - Atoms
   $$\varrho_0 := \frac{\mathbb{1}}{(2j+1)^N}$$
   - Light
   $$\langle \mathbf{S} \rangle = (S_0, 0, 0).$$
2. Measurement of J_x + feedback or postselection.
3. Measurement of J_y + feedback or postselection.
4. Measurement of J_z + feedback or postselection.

- We consider 10^6 spin-1 ^{87}Rb atoms and $S_0 = 0.5 \times 10^8$.
- Initial state of the atoms has $(\Delta J_k)^2 \sim N$ for $k = x, y, z$.
- After squeezing, we obtain $\xi_s < 1$.
- Thus, we get a state close to a singlet state.

Gaussian states

- Gaussian states are quantum states for which all third and higher order correlations are trivial.

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- Continuous variable systems: The dynamics of Gaussian systems can be followed by writing down dynamical equations for the covariance matrix and the expectation values of x_k and p_k. For a single mode, this matrix looks like

$$\Gamma_{xp} \propto \begin{pmatrix} \langle x^2\rangle - \langle x\rangle^2 & \langle xp + px\rangle/2 - \langle x\rangle\langle p\rangle \\ \langle xp + px\rangle/2 - \langle x\rangle\langle p\rangle & \langle p^2\rangle - \langle p\rangle^2 \end{pmatrix}$$

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- Spin systems: Such ideas can be used if one of the spin components is large. Then the other two components behave like x and p operators.
- We extend this approach to states for which the spin is not large. Our covariance matrix for a single spin is

$$\Gamma_J \propto \begin{pmatrix} \langle \Delta J_x \Delta J_x\rangle & \langle \Delta J_y \Delta J_x\rangle & \langle \Delta J_z \Delta J_x\rangle \\ \langle \Delta J_x \Delta J_y\rangle & \langle \Delta J_y \Delta J_y\rangle & \langle \Delta J_z \Delta J_y\rangle \\ \langle \Delta J_x \Delta J_z\rangle & \langle \Delta J_y \Delta J_z\rangle & \langle \Delta J_z \Delta J_z\rangle \end{pmatrix}.$$

Covariance matrix

- We define the set of operators

$$R = \{\frac{J_x}{\sqrt{J}}, \frac{J_y}{\sqrt{J}}, \frac{J_z}{\sqrt{J}}, \frac{S_x}{\sqrt{S}}, \frac{S_y}{\sqrt{S}}, \frac{S_z}{\sqrt{S}}\}$$

and covariance matrix as

$$\Gamma_{mn} := \langle R_m R_n + R_n R_m \rangle / 2 - \langle R_m \rangle \langle R_n \rangle.$$

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- Consider dynamics for $t \sim \tau$.
- For short times, the dynamics of an operator O_0 is given by

$$O_P = O_0 - it[O_0, H],$$

where we assumed $\hbar = 1$.

Covariance matrix II

- Dynamical equations for Γ_{kl} in terms of other correlation terms Γ_{mn} and higher order correlations. Let us make the reasonable assumption that variances stay small during squeezing

$$|\langle(\prod_{k=1}^{K} \Delta J_{a_k})(\prod_{l=1}^{L} \Delta S_{b_l})\rangle| \ll J^K S_0^L.$$

- Hence one arrives to

$$\Gamma_P = M\Gamma_0 M^T, \tag{1}$$

where M is the identity matrix, apart from $M_{5,1} = \frac{\langle S_x \rangle}{S_0}\kappa$, and $\kappa := t/\tau = \Omega t\sqrt{JS_0}$.

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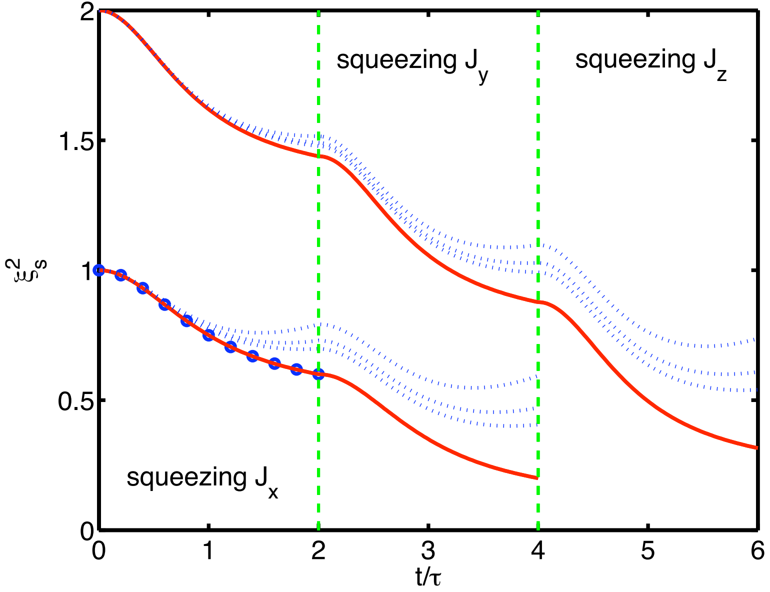
- The measurement of the light can be modeled with a projection

$$\Gamma_M = \Gamma_P - \Gamma_P(P_y\Gamma_P P_y)^{\mathrm{MP}}\Gamma_P^T, \tag{2}$$

where MP denotes the Moore-Penrose pseudoinverse, and P_y is $(0,0,0,0,1,0)$. [G. Giedke and J.I. Cirac, Phys. Rev. A **66**, 032316 (2002).]

Spin squeezing dynamics (top curve, solid)



Modeling losses

The dynamics of the covariance matric for the case of losses

$$\Gamma'_P = (\mathbb{1} - \eta D) M \Gamma_0 M^T (\mathbb{1} - \eta D) + \eta(2-\eta) D \Gamma_{\text{noise}},$$

where $D = \text{diag}(1,1,1,0,0,0)$ and $\Gamma_{\text{noise}} = \text{diag}(\frac{2}{3}, \frac{2}{3}, \frac{2}{3}, 0, 0, 0)$.

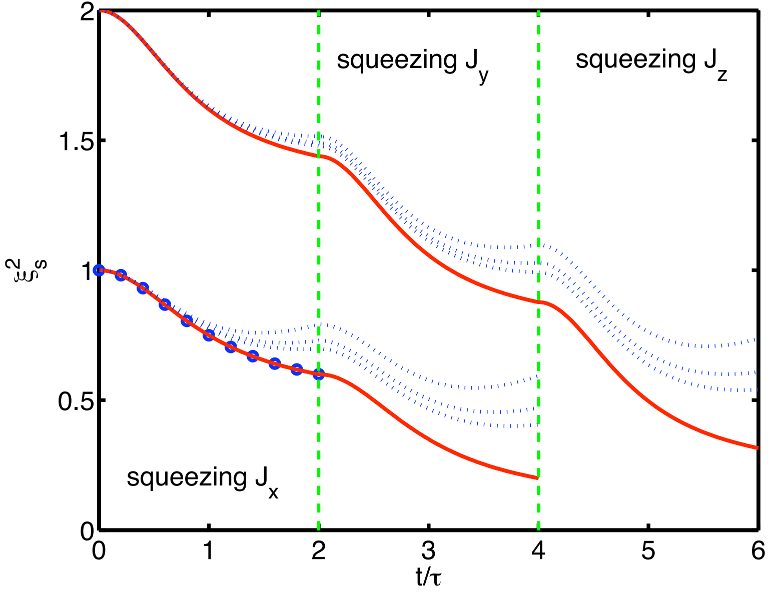
- η is the probability of spontaneous excitation by the off-resonant probe, that is, the fraction of atoms that decoherence during the QND process.
- The losses are connected to κ through

  $$\eta = Q\kappa^2/\alpha,$$

  where α is the resonant optical depth of the sample and $Q = \frac{8}{9}$

  [L.B. Madsen and K. Mølmer, Phys. Rev. A **70**, 052324 (2004).]

Spin squeezing dynamics: $\alpha = 50, 75, 100$ (dotted)



Outline

Von Neumann measurement

- Let us consider the initial state (half of the spins are in the $|+1\rangle_x$ state, half of them are in the $|-1\rangle_x$ state)

$$|\Psi\rangle_0' := |+j\rangle\langle +j|^{\otimes \frac{N}{2}} \otimes |-j\rangle\langle -j|^{\otimes \frac{N}{2}}. \tag{3}$$

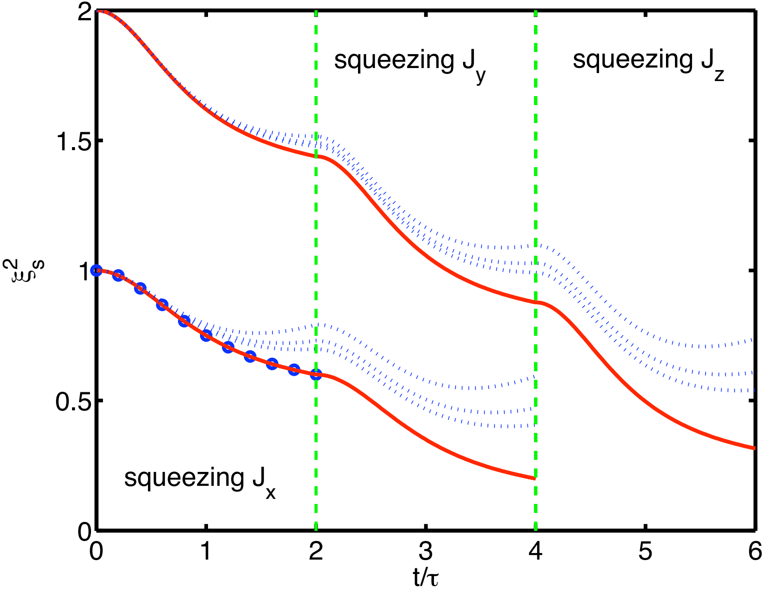
- The projective measurement of J_x on $|\Psi\rangle_0'$ leads to a mixture of product states with N_1 atoms in the $|+j\rangle_x$ state and N_2 atoms in the $|-j\rangle_x$ state. The variance of N_1 is $(\Delta N_1)^2 = \frac{N}{4}$.
- Now, a von Neumann measurement of J_y decreases its variance $(\Delta J_y)^2$ effectively to zero.
- For large N, we have $(\Delta J_x)^2 = (\Delta J_z)^2 = \frac{Nj^2}{2} + 2|N_1 - \frac{N}{2}|^2 j^2$.
- This gives $\xi_s < 1$ if $|N_1 - \frac{N}{2}|^2 < \frac{N}{4}$, and for $N_1 = \frac{N}{2}$ we obtain $\xi_s^2 = \frac{1}{2}$.
- We get squeezing in the long time limit.

Exact model

Results: for $t \sim \tau \times N^{\frac{1}{4}}$ the variances decrease to $\sim \sqrt{N}$, while for $t \sim \tau \times \sqrt{N}$ the variances reach ~ 1, which we call the von Neumann limit.

- Straightforward simulation of the quantum dynamics of million atoms is not possible.
- However, in the large N limit, a formalism can be obtained that replaces sums by integrals. Such integrals can be computed numerically or analytically.
- Thus, this approach works also for the regime in which the Gaussian approximation is no more valid.
- Comparison with exact model is possible for an initial state for which half of the spins are in the $|+1\rangle_x$ state, half of them are in the $|-1\rangle_x$ state.

Spin squeezing dynamics (bottom curve, dots)

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Conclusions

- We presented a method for creating and detecting entanglement in an ensemble of atoms with spin $j > \frac{1}{2}$.
- Our experimental proposal aims to create a many-body singlet state through squeezing the uncertainties of the collective angular momenta.
- We showed how to use an extension of the usual Gaussian formalism for modeling the experiment.
- Presentation based on: GT, M.W. Mitchell, in preparation; soon on the arxiv.

*** THANK YOU ***