Permutationally invariant quantum tomography and state reconstruction

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EUSKO JAURLARITZA GOBIERNO VASCO

Outline

1. **Motivation**
   - Why quantum tomography is important?
2. **Quantum experiments with multi-qubit systems**
   - Physical systems
   - Local measurements
3. **Full quantum state tomography**
   - Basic ideas and scaling
   - Experiments
4. **Permutationally invariant tomography and state reconstruction**
   - Permutationally invariant tomography
   - Permutationally invariant state reconstruction
   - Experiment with six qubits

Why tomography is important?

- Many experiments aim to create many-body entangled states.
- Quantum state tomography is used to check the state prepared.
- The number of measurements scales exponentially with the number of qubits.

Outline

Physical systems

State-of-the-art in experiments

- 14 qubits with trapped cold ions
  T. Monz, P. Schindler, J.T. Barreiro, M. Chwalla, D. Nigg, W.A. Coish, M. Harlander, W. Haensel, M. Hennrich, R. Blatt, Phys. Rev. Lett. 106, 130506 (2011).

- 10 qubits with photons
  W.-B. Gao, C.-Y. Lu, X.-C. Yao, P. Xu, O. Gühne, A. Goebel, Y.-A. Chen, C.-Z. Peng, Z.-B. Chen, J.-W. Pan, Nature Physics, 6, 331 (2010).

Outline

Only local measurements are possible

Definition

A single local measurement setting is the basic unit of experimental effort.

A local setting means measuring operator $A^{(k)}$ at qubit k for all qubits.

$A^{(1)}$	$A^{(2)}$	$A^{(3)}$	...	$A^{(N)}$

- All two-qubit, three-qubit correlations, etc. can be obtained.

$$\langle A^{(1)}A^{(2)}\rangle, \langle A^{(1)}A^{(3)}\rangle, \langle A^{(1)}A^{(2)}A^{(3)}\rangle ...$$

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Full quantum state tomography

- The density matrix can be reconstructed from 3^N measurement settings.

Example

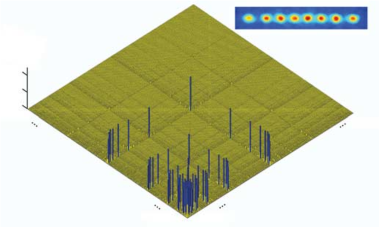
For $N = 4$, the measurements are

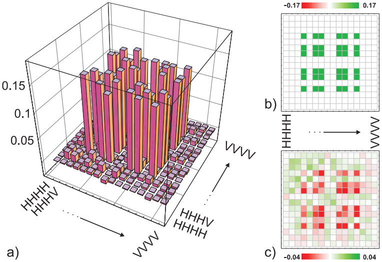
1.	X	X	X	X
2.	X	X	X	Y
3.	X	X	X	Z
		...		
3^4.	Z	Z	Z	Z

- Note again that the number of measurements scales exponentially in N.

Outline

Experiments with ions and photons





- H. Haeffner, W. Haensel, C. F. Roos, J. Benhelm, D. Chek-al-kar, M. Chwalla, T. Koerber, U. D. Rapol, M. Riebe, P. O. Schmidt, C. Becher, O. Gühne, W. Dür, R. Blatt, Nature 438, 643-646 (2005).
- N. Kiesel, C. Schmid, G. Tóth, E. Solano, and H. Weinfurter, Phys. Rev. Lett. 98, 063604 (2007).

Alternative approaches (alphabetical order)

- Compressed sensing: Low rank states
  D. Gross, Y.-K. Liu, S.T. Flammia, S. Becker, and J. Eisert, Phys. Rev. Lett. 105, 150401 (2010).
  - Low rank states of any type
- MPS tomography: If the state is expected to be of a certain form, we can measure the parameters of the ansatz.
  M. Cramer, M.B. Plenio, S.T. Flammia, R. Somma, D. Gross, S.D. Bartlett, O. Landon-Cardinal, D. Poulin and Yi.K. Liu, Nature Communications 1, Article number: 149 (2010).
  - Spin chain states
- PI tomography: Tomography in a subspace of the density matrices (our approach)
  G. Tóth, W. Wieczorek, D. Gross, R. Krischek, C. Schwemmer, and H. Weinfurter, Phys. Rev. Lett. 105, 250403 (2010).
  - Permutationally invariant states (not only symmetric states)

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Permutationally invariant tomography

PRL **105,** 250403 (2010) PHYSICAL REVIEW LETTERS week ending 17 DECEMBER 2010

Permutationally Invariant Quantum Tomography

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We present a scalable method for the tomography of large multiqubit quantum registers. It acquires information about the permutationally invariant part of the density operator, which is a good approximation to the true state in many relevant cases. Our method gives the best measurement strategy to minimize the experimental effort as well as the uncertainties of the reconstructed density matrix. We apply our method to the experimental tomography of a photonic four-qubit symmetric Dicke state.

DOI: 10.1103/PhysRevLett.105.250403 PACS numbers: 03.65.Wj, 03.65.Ud, 42.50.Dv

Because of the rapid development of quantum experiments, it is now possible to create highly entangled multiqubit states using photons [1–5], trapped ions [6], and cold atoms [7]. So far, the largest implementations that allow for an individual readout of the particles involve on the order of 10 qubits. This number will soon be overcome, for example, by using several degrees of freedom within each particle to store quantum information [8]. Thus, a new regime will be reached in which a complete state tomography is impossible even from the point of view of the storage place needed on a classical computer. At this point the question arises: Can we still extract useful information

for both density matrices and are thus obtained exactly from PI tomography [2–4]. Finally, if $\varrho_{\rm PI}$ is entangled, so is the state ϱ of the system, which makes PI tomography a useful and efficient tool for entanglement detection.

Below, we summarize the four main contributions of this Letter. We restrict our attention to the case of N qubits—higher-dimensional systems can be treated similarly.

(1) In most experiments, the qubits can be individually addressed whereas nonlocal quantities cannot be measured directly. The experimental effort is then characterized by the number of local measurement settings needed, where ''setting'' refers to the choice of one observable per qubit,

Permutationally invariant part of the density matrix

Permutationally invariant part of the density matrix:

$$\varrho_{\rm PI} = \frac{1}{N!}\sum \Pi_k \varrho \Pi_k^\dagger,$$

where Π_k are all the permutations of the qubits.

- Related literature: Reconstructing $\varrho_{\rm PI}$ for spin systems.

[G. M. D'Ariano *et al.*, J. Opt. B **5**, 77 (2003).]

- Photons in a single mode optical fiber are always in a permutationally invariant state. Small set of measurements are needed for their characterization (experiments).

[R.B.A. Adamson *et al.*, Phys. Rev. Lett. **98**, 043601 (2007); R.B.A. Adamson *et al.*, Phys. Rev. A 2008; L. K. Shalm *et al.*, Nature **457**, 67 (2009).]

Meaning of the PI part of the density matrix

- The PI part of the density matrix is meaningful, even if the density matrix is far from being permutationally invariant.
- It is the quantum state we get after we forget how we labeled the particles.

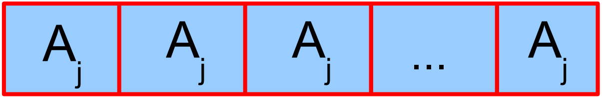
Main features of the method

Features of our method:

1. Is for spatially separated qubits.
2. Needs the minimal number of measurement settings.
3. Uses the measurements that lead to the smallest uncertainty possible of the elements of $\varrho_{\rm PI}$.
4. Gives an uncertainty for the recovered expectation values and density matrix elements.
5. If $\varrho_{\rm PI}$ is entangled, so is ϱ. Can be used for entanglement detection!
6. Expectation value of permutationally invariant operators can be obtained exactly (i.e., fidelity to Dicke states).

Measurements

- We measure the same observable A_j on all qubits. (Necessary for optimality.)



- Each qubit observable is defined by the measurement directions $\vec{a}_j$ using $A_j = a_{j,x}X + a_{j,y}Y + a_{j,z}Z$.

Number of measurement settings:

$$\mathcal{D}_N = \binom{N+2}{N} = \frac{1}{2}(N^2 + 3N + 2).$$

What do we get from the measurements?

We obtain the expectation values for

$$\langle (A_j^{\otimes (N-n)} \otimes \mathbb{1}^{\otimes n})_{\mathrm{PI}} \rangle$$

for $j = 1, 2, .., \mathcal{D}_N$ and $n = 0, 1, ..., N$.

How do we obtain operator expectation values?

A Bloch vector element can be obtained as

$$\underbrace{\langle (X^{\otimes k} \otimes Y^{\otimes l} \otimes Z^{\otimes m} \otimes \mathbb{1}^{\otimes n})_{\mathrm{PI}} \rangle}_{\text{Bloch vector elements}} = \sum_{j=1}^{\mathcal{D}_N} \underbrace{c_j^{(k,l,m)}}_{\text{coefficients}} \times \underbrace{\langle (A_j^{\otimes (N-n)} \otimes \mathbb{1}^{\otimes n})_{\mathrm{PI}} \rangle}_{\text{Measured data}}.$$

- From the Bloch vector elements, the density matrix can be reconstructed.
- Expectation values of all PI operators can be obtained.
- Uncertainties can also be obtained assuming Gaussian statistics.

Optimization for A_j

- We have to find the measurement operators minimizing

$$(\mathcal{E}_{\mathrm{total}})^2 = \sum_{k+l+m+n=N} \mathcal{E}^2\left[(X^{\otimes k} \otimes Y^{\otimes l} \otimes Z^{\otimes m} \otimes \mathbb{1}^{\otimes n})_{\mathrm{PI}}\right] \times \left(\frac{N!}{k!l!m!n!}\right).$$

How much is the information loss?

Estimation of the fidelity $F(\varrho, \varrho_{\rm PI})$:

$$F(\varrho, \varrho_{\rm PI}) \geq \langle P_{\rm s} \rangle_{\varrho}^2 \equiv \langle P_{\rm s} \rangle_{\varrho_{\rm PI}}^2,$$

where $P_{\rm s}$ is the projector to the N-qubit symmetric subspace.

- $F(\varrho, \varrho_{\rm PI})$ can be estimated only from $\varrho_{\rm PI}$!

4-qubit Dicke state, optimized settings (exp.)

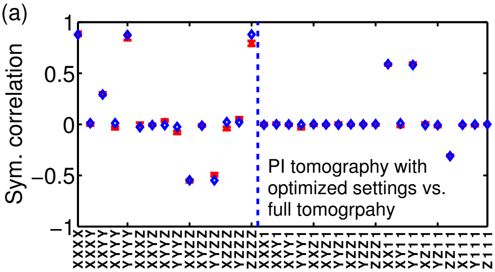
- The symmetric Dicke state with $j_z = 0$ is

$$|j = \frac{N}{2}, j_z = 0\rangle = \binom{n}{N}^{-\frac{1}{2}} \sum_k \mathcal{P}_k (|+\frac{1}{2}\rangle^{\otimes N/2} |-\frac{1}{2}\rangle^{\otimes N/2}),$$

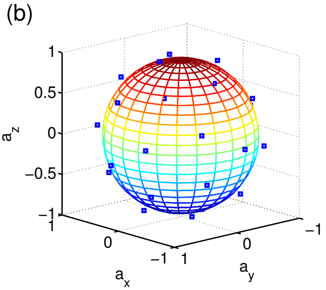
  where the summation is over all distinct permutations.

- Experiment for $N = 4$.

4-qubit Dicke state, optimized settings (exp.) II

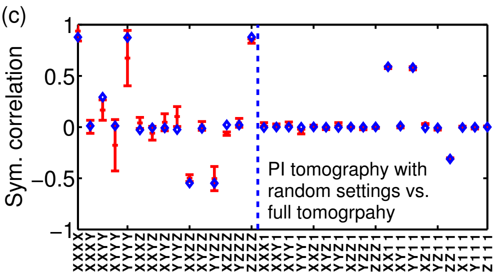


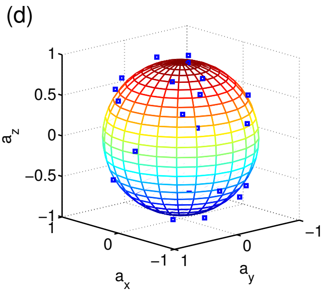
The measured correlations



$\vec{a}_j$ measurement directions

Random settings (exp.)

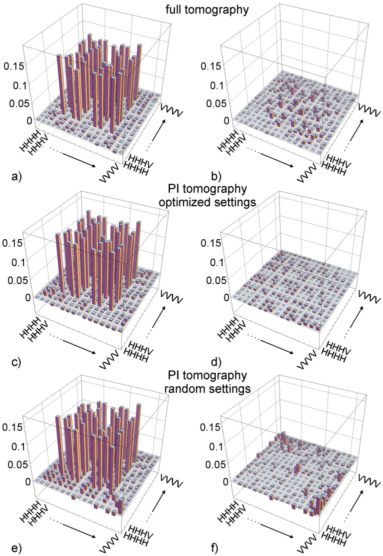




The measured correlations

$\vec{a}_j$ measurement directions

Density matrices (exp.)



Outline

Permutationally invariant state reconstruction

New Journal of Physics
The open-access journal for physics

Permutationally invariant state reconstruction

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How do we get the density matrix?

Semi-scalable fitting

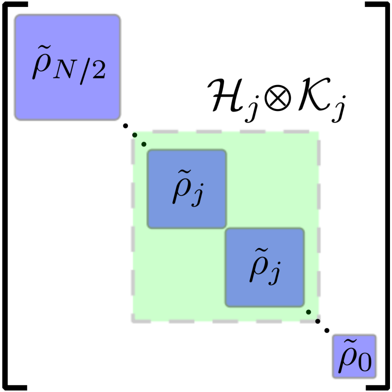
- Simple idea:
  - 1. Reconstruct all Bloch vector elements.
  - 2. Reconstruct the density matrix.
  - 3. Find the physical matrix by fitting.
- Problem: the physical matrix does not fit into the computer.

- Solution: another representation of the density matrix.

Scalable fitting of a physical state

- The alternative representation of the PI matrix is

$$\rho_{\mathrm{PI}} =$$



- All blocks must be physical (unnormalized) density matrices.

Fitting methods and results

- Fit functions:

Table 1. Common reconstruction principles and their corresponding fit functions $F(\rho)$ used in the optimization given by equation (4); see text for further details.

Reconstruction principle	Fit function $F(\rho)$
Maximum likelihood [23]	$-\sum_k f_k \log[p_k(\rho)]$
Least squares [24]	$\sum_k w_k[f_k - p_k(\rho)]^2,\ w_k > 0$
Free least squares [4]	$\sum_k 1/p_k(\rho)[f_k - p_k(\rho)]^2$
Hedged maximum likelihood [25]	$-\sum_k f_k \log[p_k(\rho)] - \beta \log[\det(\rho)],\ \beta > 0$

- Run time for up to 20 qubits:

Table 2. Current performance of the convex optimization algorithm on the described test procedure and on frequencies from simulated experiments; free least squares provides similar results to the maximum likelihood principle.

	$N = 8$	$N = 12$	$N = 16$	$N = 20$
Maximum likelihood				
Algorithm test	8.5 s	47 s	2.7 min	9.2 min
Simulated experiment	9.2 s	48 s	2.9 min	9.3 min
Least squares				
Algorithm test	8.4 s	39 s	2.5 min	6 min
Simulated experiment	9.2 s	43 s	2.7 min	6.7 min

Fitting methods and results II

- Guaranteed to find the global optimum.
- Fast: before, the time for fitting was a bottleneck of full tomography.

Outline

Experiment with the Six Qubit Symmetric Dicke State (DPG 2012, Stuttgart)

Q: Fachverband Quantenoptik und Photonik

Q 8: Quanteninformation: Konzepte und Methoden 2

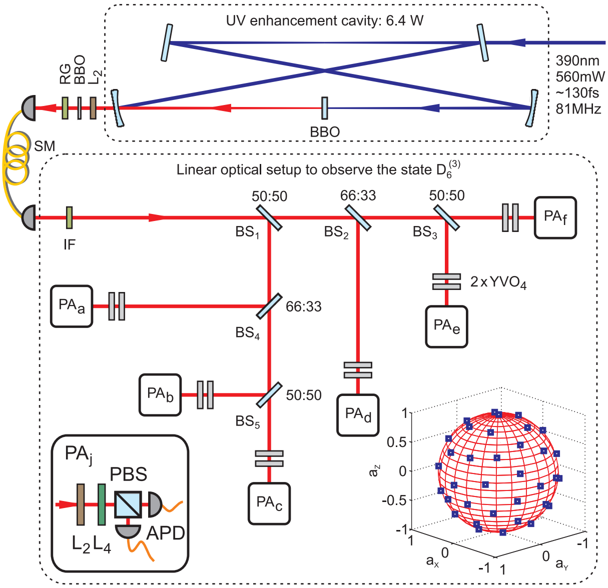
Q 8.7, Mon, 03.30 PM–03.45 PM, V38.04

Permutationally Invariant Tomography of a Six Qubit Symmetric Dicke State — •Christian Schwemmer[1,2], Géza Tóth[3,4,5], Alexander Niggebaum[1,2], Tobias Moroder[6], Philipp Hyllus[3], Otfried Gühne[6,7], and Harald Weinfurter[1,2] — [1]MPI für Quantenoptik, D85748 Garching — [2]Fakultät für Physik, LudwigMaximiliansUniversität, D80797 München — [3]Department of Theoretical Physics, The University of the Basque Country, E48080 Bilbao — [4]IKERBASQUE, Basque Foundation for Science, E48011 Bilbao — [5]Research Institute for Solid State Physics and Optics, Hungarian Academy of Sciences, H1525 Budapest — [6]Institut für Quantenoptik und Quanteninformation, Österreichische Akademie der Wissenschaften, A6020 Insbruck — [7]NaturwissenschaftlichTechnische Fakultät, Universität Siegen, D57072 Siegen,

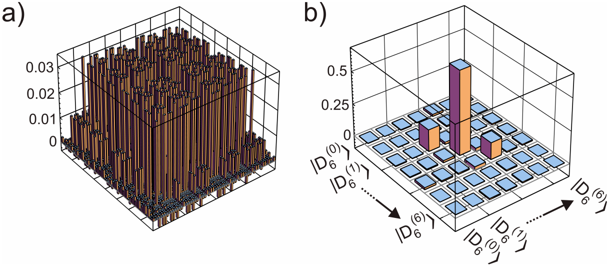
Multipartite entangled quantum states are promising candidates for potential applications like quantum metrology or quantum communication. Yet, efficient tools are needed to characterize these states and to evaluate their applicability. Standard quantum state tomography suffers from an exponential increase in the measurement effort with the number of qubits. Here, we show that by restricting to permutational invariant states like GHZ, W or symmetric Dicke states the problem can be recast such that the measurement effort scales only quadratically [1]. We apply this method to experimentally analyze a six photon symmetric Dicke state generated by parametric down conversion where instead of 729 only 28 basis settings have to be measured.

[1] Tóth et al., Phys. Rev. Lett. 105, 250403 (2010).

Experimental setup



Results



- Most of the noise comes from the two “neighboring” Dicke states with one excitation more and one excitation fewer.

Application: PPT mixer

- L. Novo, T. Moroder and O. Gühne: detects genuine multipartite entanglement in PI states (work in progress).

Summary

- PI tomography and state reconstruction is a fully scalable reconstruction scheme.
- No assumptions are needed to get a correct output.
- These pave the way for quantum experiments with more than 6 – 8 qubits.

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