EXPERIMENTAL COMPARISON OF EFFICIENT TOMOGRAPHY SCHEMES FOR A SIX QUBIT STATE (SUPPLEMENTAL MATERIAL)

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In the Supplemental Material, we present further experimental results and calculations.

THE SETUP

The photon source is based on a femtosecond enhancement cavity in the UV with a 1 mm thick β-barium borate (BBO) crystal cut for type II phase matching placed inside [22] (Fig. S1). In order to compensate for walk off effects a half-wave plate (HWP) and a second BBO crystal of 0.5 mm are applied. Spatial filtering is achieved by coupling the photons into a single mode fiber (SM) and an interference filter (IF) ($\Delta\lambda = 3$ nm) enables spectral filtering. Distributing the photons into six spatial modes is realized by 3 beam splitters with a splitting ratio of 50:50 (BS_1, BS_3, BS_4) and two beam splitters with a ratio of 66:33 (BS_2, BS_4). Yttrium-vanadate (YVO_4) crystals are used to compensate for unwanted phase shifts. State analysis is realized by half-wave and quarter-wave plates (QWP) and polarizing beam splitters (PBS). The photons are detected by fiber-coupled single photon counting modules connected to a FPGA-based coincidence logic.

In Fig. S1 (lower right corner) a visualization of the measurement directions on the Bloch sphere is depicted. Each point (a_x, a_y, a_z) on the sphere corresponds to a measurement operator of the form $a_x\sigma_x + a_y\sigma_y + a_z\sigma_z$. In order to perform PI tomography for 6 qubits 28 operators have to be measured.

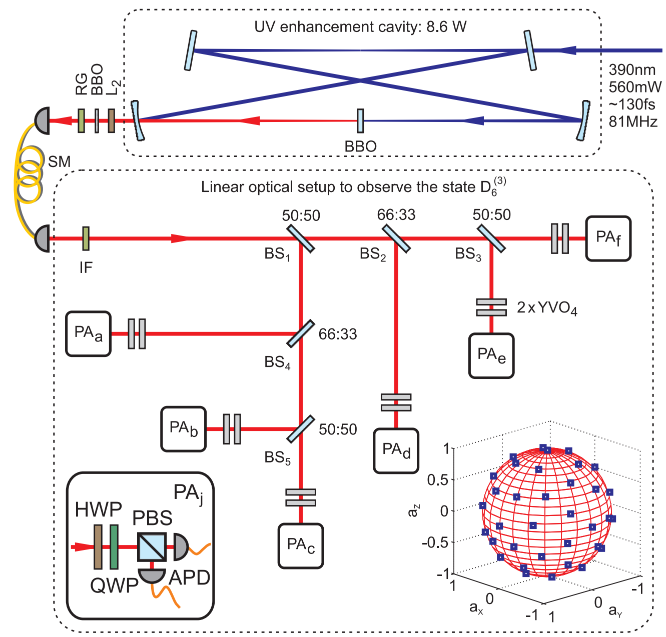

Figure S1. Schematic drawing of the experimental setup to observe the symmetric Dicke state $|D_6^{(3)}\rangle$ For a description, see text.

STATE RECONSTRUCTION

The target function to be minimized is the *logarithmic likelihood* which is given by $\sum_{k,s} \frac{n_{k,s}}{N_{\max}} \log(p_{k,s})$ where $n_{k,s}$ labels the number of counts for the outcome k when measuring setting s with the corresponding probability $p_{k,s}$ for the guess $\hat{\varrho}$. In order to take into account slightly different total count numbers per setting, the $n_{k,s}$ have to be divided by the maximum count number observed in one setting $N_{\max} = \max(N_s)$.

For CS exactly the same target function has to be minimized with the only difference that the underlying set of measurement data is tomographically incomplete.

CONVERGENCE OF CS IN THE PI SUBSPACE

As described in the main text, we performed PI tomography together with CS in the PI subspace at different UV pump powers. In order to investigate the convergence of CS, series of different samples were randomly chosen from the full set of measurements. For all pump powers, the average fidelity with respect to all PI settings is above 0.950 as soon as the number of settings is ≥ 12 (out of 28), see Fig. S2.

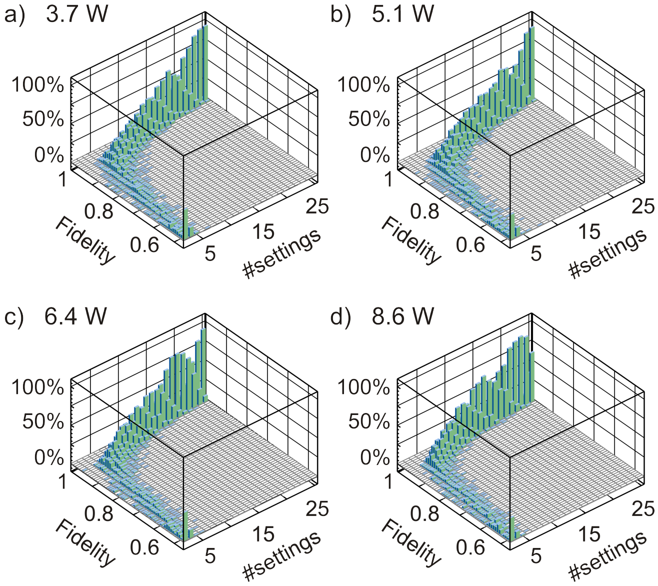

Figure S2. Probability to observe a certain fidelity for arbitrarily chosen tomographically incomplete sets of settings in comparison with PI tomography from 28 settings at different pump levels. As soon as the number of settings surpasses 12, the state is almost perfectly determined, i.e., the overlap with respect to the states reconstructed from all settings ≥ 0.950.

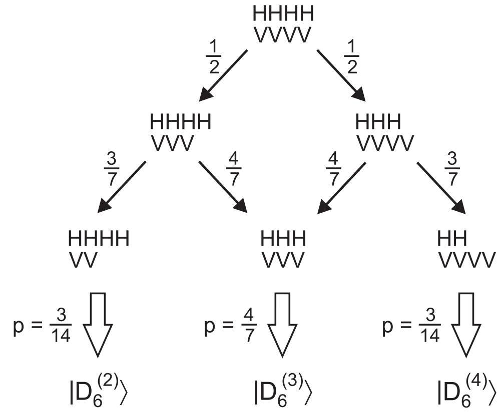

Figure S3. The loss of two photons in a 8 photon event leads to an admixture of the state $\varrho_{D_6^{(2)}}$ and $\varrho_{D_6^{(4)}}$ to the target state. The respective probabilities p can be determined by simple combinatorics.

NOISE MODEL

As already explained in the main part of this paper, SPDC is a spontaneous process and therefore with a certain probability 8 photons are emitted from the source. The loss of two of these 8 photons in the linear optical setup and subsequent detection leads to an admixture of the states $\varrho_{D_6^{(2)}}$ and $\varrho_{D_6^{(4)}}$ for the case that either two H or two V polarized photons are not detected, respectively. However, in the case that one H and one V polarized photon remain undetected a considerable amount of this higher-order noise consists of the target state $\varrho_{D_6^{(3)}}$ thus preserving genuine multipartite entanglement even at high UV pump powers. The probabilities of the respective states to occur can be deduced from simple combinatorics, see Fig. S3. From this simple noise model, an experimental state of the form

$$\varrho_{\mathrm{exp}}^{\mathrm{noise}}(q,\lambda) = (1-q)\varrho_{D_6^{(3)}} + q\varrho_6 \tag{S1}$$

with

$$\varrho_6 = \frac{4}{7}\varrho_{D_6^{(3)}} + \frac{3}{14}\left[\varrho_{D_6^{(2)}} + \varrho_{D_6^{(4)}}\right] \tag{S2}$$

would be expected. However, this is not observed experimentally since the emission angles of down-conversion photons are polarization dependent [26, 27] leading to an asymmetry in the coupling into the single mode fiber used. Therefore, the noisemodel was extended by the asymmetry parameter λ. Both q and λ can be deduced form the fidelities F with respect to the Dicke states $|D_6^{(2)}\rangle$, $|D_6^{(3)}\rangle$ and $|D_6^{(4)}\rangle$

$$\begin{aligned} q &= \frac{7}{3}\cdot\frac{F_{|D_6^{(2)}\rangle} + F_{|D_6^{(4)}\rangle}}{F_{|D_6^{(2)}\rangle} + F_{|D_6^{(3)}\rangle} + F_{|D_6^{(4)}\rangle}}, \\ \lambda &= \frac{F_{|D_6^{(2)}\rangle} - F_{|D_6^{(4)}\rangle}}{F_{|D_6^{(2)}\rangle} + F_{|D_6^{(4)}\rangle}}. \end{aligned} \tag{S3}$$

ENTANGLEMENT WITNESS

Entanglement witnesses with respect to symmetric states are PI operators and thus can be determined efficiently. For detecting genuine multipartite entanglement, we used the entanglement witness

$$\begin{aligned} \mathcal{W} = 0.420 \cdot \mathbb{1} &- 0.700|D_6^{(3)}\rangle\langle D_6^{(3)}| \\ &- 0.160|D_6^{(2)}\rangle\langle D_6^{(2)}| - 0.140|D_6^{(4)}\rangle\langle D_6^{(4)}|, \end{aligned} \tag{S4}$$

where an expectation value $\langle\mathcal{W}\rangle < 0$ rules out any biseparability. In order to obtain $\mathcal{W}$ we take an operator of the form

$$\begin{aligned} A_\alpha = \alpha|D_6^{(3)}\rangle\langle D_6^{(3)}| &+ \beta|D_6^{(2)}\rangle\langle D_6^{(2)}| \\ &+ (1-\alpha-\beta)|D_6^{(4)}\rangle\langle D_6^{(4)}|. \end{aligned} \tag{S5}$$

An entanglement witness can be obtained as

$$W_\alpha = \max_{PPT}\langle A_\alpha\rangle \cdot \mathbb{1} - A_\alpha \tag{S6}$$

where the maximum for bipartite PPT states can be obtained with semidefinite programming [17]. For $\alpha = 0.700, \beta = 0.160$ we have for PPT states over all partitions $\max_{PPT}\langle A_\alpha\rangle = 0.420$. It is important that semidefinite programming always finds the global optimum. A systematic generalization to construct witnesses for Dicke states can be found in Ref. [18].

Here, we want use this witness to test whether, in spite of the higher-order noise, the observed states are still genuine six-partite entangled. For the corresponding pump powers from 3.7 W to 8.6 W, we determined the expectation value of $\mathcal{W}$ as -0.088 ± 0.006, -0.078 ± 0.006, -0.075 ± 0.006 and -0.048 ± 0.005 for PI tomography and -0.082 ± 0.011, -0.064 ± 0.013, -0.083 ± 0.009 and -0.044 ± 0.009 for CS in the PI subspace. Clearly, due to the high probability of $\varrho_{D_6^{(3)}}$ states in the higher-order noise the entanglement is maintained also for high pump powers.